LA PRIMA PARTE DEL GENERAL TRATTATO DI NUMERI, ET MISURE BOOK 16, SECTION 206

NICOLÒ TARTAGLIA

INTRODUCTION

Here we present the relevant text of the *General Trattato* of Tartaglia [2] associated with the problem of points and a paraphrase of it into English. As far as I have been able to determine, the only translation into a modern language is a rendering into German which appears in the anthology of Ivo Schneider [1]. I acknowledge a debt to it.

An image of the text is publicly available at ECHO (the European Cultural Heritage Online)[1]

Tartaglia himself offers a paraphrase of Pacioli. It is interesting to compare the texts.

TEXT AND ENGLISH PARAPHRASE

Error di fra Luca dal Borgo

(206) Frate Luca dal Borgo mette una simil questione.

Una brigata giuocano alla balla a 60 al giuoco, & a 10 per caccia, & fanno la posta ducati 22 accade certi accidenti, che non ponno compir il giuoco, & una parte ha 50 & 'altra ha 30. Si adimanda che toccata per parte di detta posta. In questa tal questione dice il detto fra Luca, che l'ha trovato di diverse openioni, si in un lato, come nell'altro, ma che tutti loro argumenti gli pareno frasche, & dice che la retta via, & verità è questa, che tal ragione si puo far in tre modi.

La prima dice che si debbe considerare quante cazze al piu si possino fare fra una parte, & l'altra, & che si trovara esser 11, cioe quando sono a 50 per uno, & cosi dice, che'l si vede, che quello da 50 ne ha li 5 undecimi

(206) Brother Luca of Borgo submits a similar problem:

A company plays ball to 60 for the game, and with 10 for a goal. And they make the stake 22 Ducats. It happens by certain accidents they cannot complete the game to the end; but one party has 50 and the other has 30. One asks, what is due to each party of the said stake. In this such question the named brother Luca says, that he has found diverse opinions, but on one side as on the other, but that to him all their arguments appear confused, and that the right way and the truth is the one that such reasoning is able to be made in three ways.

The first he says that one must consider, how many goals could be made at most by one and the other party, and that is found to be 11, that is when each has 50, and thus he says, that one sees, that

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Date: Venice, 1556.

English translation by Richard Pulskamp, Department of Mathematics, Xavier University, Cincinnati, OH. Created on July 18, 2009.

[1] `http://echo.mpiwg-berlin.mpg.de/content`, Band I, images 544 and 545.

di queste cazze, & che quello da 30 ne ha li 3 undecimi,

E pero dice, che una parte si debbe tirar li 5 undecimi delli detti ducati 22 & l'altra ne debbe tirare per 3 undecesimi, che summati insieme fanno 8 undecimi, dapoi dice, che si debba procedere per modo di compagnia dicendo, se 8 undececimi guadagna ducati 22 che guadagnara 5 undecimi, & 3 undecimi, onde operando si trovara che a quello di 50 gli toccara ducati 13, & 3 quarto & a quello di 30 li ne toccara 8 & 1 quart.

Laqual sua regola a me non pare, ne bella, ne buona, perche se per forte una delle parti havesse 10 & l'altra havesse nulla, procedendo per tal sua regola seguiria, che quella parte, che havesse il detto 10 doveria tirar il tutto, & l'altra non doveria tirar cosa alcuna, che saria in tutto fuora di ragione, che per haver 10 dovesse tirar il tutto.

E per tanto dico, che la risolutione di una tal questione è piu presto giudiciale, che per ragione, tal che in qual si voglia modo la fara risolta vi si trovara da litigare, nondimeno il men litigioso, a me par, che sia questo, prima si debbe vedere, che parte ha ciascun di tutto il giuoco, cioe se per forte uno havesse 10, & l'altro 0 adunque colui, che ha 10 haveria il sesto di tutto il giuoco, e per tanto dico, che in questo caso, doveria haver la sesta parte delli denari, che metteno per uno, cioe si metton ducati 22 per parte, lui doveria haver la sesta parte di detti ducati 22 che faria ducati 3 e doi 3 che gióti cō li suo ducati 22 fariano ducati 25 e doi 3 & l'altra parte dovera tirar il resto, il qual resto faria ducati 18 e un 3. Et se una parte havesse 50 & l'altra 30 cava 30 di 50 restara 20 il qual 20 vien a esser il terzo di tutto il giuoco, e pero dovera tirar, oltra li suoi, la terza parte delli danari dell'altra parte, laqual terza parte faria ducat 7 e un 3

the one with 50 has due 5 elevenths of these goals and that the one with 30 has 3 elevenths.

And next he says, that a party should take 5 elevenths of the aforementioned 22 ducats and the other should take 3 elevenths; this sum makes altogether 8 elevenths; furthermore he says, that one must proceed as in a trading company, if 8 elevenths is worth 22 ducats what is 5 elevenths and 3 elevenths worth, so that one proceeds so to find that to the one with 50 is due 13 and 3 quarter ducats and to the one with 30 is due 8 and 1 quarter.

This rule of his seems to me to be neither beautiful nor good, because if perchance one of the parties had 10 and the other none, one would proceed according to his rule, that that party which had the said 10 should take all, and the other should take nothing, which is completely beyond reason, that one with 10 could take the whole.

And therefore I say, that the resolution of such a problem is rather judicial than through computation, because in whatever manner through which it is resolved to you, there will be found arguing. Nevertheless men argue, it seems to me, this or that. First one must see what part each has of the complete game, that is, if one perchance had 10 and the other 0, thus the who has 10, will have the sixth of the complete game; and therefore I say, that in this case, he should receive the sixth part of the money that each have put; that is, if they stake 22 ducats per party, he should have the sixth part of the said 22 ducats, which makes 3 and two-thirds, the total with his 22 ducats makes 25 and two-thirds, and the other party may take the rest, this remainder is 18 and one-third ducats. And if one party had 50 and the other 30,

che cō li suoi faria ducati 29 e un 3 & l'altra parte doveria tirar il resto, che faria ducati 14 e doi 3 & cosi procedēdo nō si trovara sequir cosa non conveniente, come fece in quella di fra Luca.

one should deduct 30 from 50. There will remain 20, and these 20 come to be one-third of the whole game. And however one should take (besides his own) one-third part of the money of the other party, which third part makes 7 and one-third ducats, that with his own will make 29 and one-third ducats. And the other party may take the rest, which will make 14 and two-thirds ducats. And proceeding so, nothing inconvenient will be found to follow, as made in the solution of brother Luca.

Gli altri duoi modi a dutti dal detto fra Luca l'uno è simile al sopranotato, anchor che in parole paia differente, & similmente il terzo, perche nel terzo vuol, che si summi 50 con 30 fa 80 poi dir se 80 mi da ducati 22 che mi dara 50 & che mi dara 30 & suppone, che li ducati 22 sia la summa di quello, che ha posto ambedue le parti, cioe che ciascuna parte habbia posto suso ducati 11 laqual solutione patira la medesima oppositione da noi adutta sopra la prima, & perche sono tai questioni materie litigose, & di poco sugo non se ne debbe tener gran conto. Due altre quasi simili consequentemente mette il detto fra Luca, lequali per esser materie di puoco sugo, & di letigo assai, mi è parso di non parlarne, abenche molti hanno da caro simili facecie per haver occasion di poter contrastare, ma che pur ne vorra ne potra formar da se, & con questo voglio facciamo fine a questo capo, & a questo libro.

One of the other two ways proposed of the aforementioned brother Luca is similar to the above described solution, even though in words slightly different, & the third is similar, because in the third solution he wants that one add 50 with 30, it makes 80. Next he says, if 80 gives to me 22 ducats, what will give to me 50 & what will give 30. And he supposes that the 22 ducats is the sum of those which are stakes of both parties, that is, that each party will have staked 11 ducats. This solution will suffer the same objections from us, that I have adopted against the first, and because these problems are matters of litigation and of little substance one need not hold them to great account. The aforementioned brother Luca sets two other nearly similar achievements, which seem to be matters of little substance and of much argument, he appears to not speak to mine, although many have from similar costly appearances in order to have occasion of being able to contrast, but that also he will be able neither to want nor to be able to form from it, and with this I wish to make to end this chapter, and to this book.

TARTAGLIA'S METHOD

Tartaglia considers a contest between two people. Suppose Player A has achieved a points, Player B b and that a total of c points are required to win. Let each ante S so that the entire stake is $2S$. According to his methodology, Player A is due $S + \frac{a-b}{c}S$ and Player B is due $S + \frac{b-a}{c}S$. That is, each player is due to gain or to be reduced by an amount proportional to the difference in the number of points achieved by each.

In the example presented, $a = 50$, $b = 30$, $c = 60$ and $S = 22$. Therefore Player A receives $22 + \frac{50-30}{60} \times 22 = 29\frac{1}{3}$ and Player B $44 - 29\frac{1}{3} = 14\frac{2}{3}$.

Pacioli's second problem concerns a contest among three people. The method of Tartaglia does not generalize to this situation.

REFERENCES

[1] Ivo Schneider. *Die Entwicklung der Wahrscheinlichkeitstheorie von den Anfängen bis 1933*. Wissenschaftliche Buchgesellschaft, Darmstadt, 1988.

[2] Niccoló Tartaglia. *La Prima parte del General Trattato di numeri et misure*. C. Troiano di Navo, Venice, 1556.