

**FROM**  
**THE DOCTRINE OF CHANCES, 3RD. ED., PP. 36–38**

ABRAHAM DE MOIVRE

PROBLEM III.

*To find in how many Trials an Event will probably happen, or how many Trials will be necessary to make it indifferent to lay on its Happening or Failing; supposing that  $a$  is the number of Chances for its happening in any one Trial, and  $b$  the number of Chances for its failing.*

SOLUTION.

Let  $x$  be the number of Trials; then by the 16th. Art. of the Introd.  $b^x$  will represent the number of Chances for the Event to fail  $x$  times successively, and  $(a + b)^x$  the whole number of Chances for happening or failing, and therefore  $\frac{b^x}{(a+b)^x}$  represents the probability of the Event's failing  $x$  times together: but by supposition that Probability is equal to the probability of its happening once at least in that number of Trials; wherefore either of those two Probabilities may be expressed by the fraction  $\frac{1}{2}$ : we have therefore the Equation  $\frac{b^x}{(a+b)^x} = \frac{1}{2}$ , or  $(a + b)^x = 2b^x$ , from whence is deduced the Equation  $x \log b = x \log b + \log 2$ ; and therefore  $x = \frac{\log 2}{\log (a+b) - \log b}$ .

Moreover, let us reassume the Equation  $(a + b)^x = 2b^x$ , wherein let us suppose that  $a, b :: 1, q$ ; hence the said Equation will be changed into this  $\left(1 + \frac{1}{q}\right)^x = 2$ . Or  $x \times \log\left(1 + \frac{1}{q}\right) = \log 2$ . In this Equation, if  $q$  be equal to 1,  $x$  will likewise be equal to 1; but if  $q$  differs from Unity, let us in the room of  $\log\left(1 + \frac{1}{q}\right)$  write its value expressed in a Series; viz.

$$\frac{1}{q} - \frac{1}{2qq} + \frac{1}{3q^3} - \frac{1}{4q^4} + \frac{1}{5q^5} - \frac{1}{6q^6}, \text{ \&c.}$$

We have therefore the Equation  $\frac{x}{q} - \frac{x}{2qq}, \text{ \&c.} = \log 2$ . Let us now suppose that  $q$  is infinite, or pretty large in respect to Unity, and then the first term of the Series will be sufficient; we shall therefore have the Equation  $\frac{x}{q} = \log 2$ , or  $x = q \log 2$ . But it is to be observed in this place that the Hyperbolic, not the Tabular, Logarithm of 2, ought to be taken, which being 0.693, &c. or 0.7 nearly, it follows that  $x = 0.7q$  nearly.

Thus we have assigned the very narrow limits within which the ratio of  $x$  to  $q$  is comprehended; for it begins with unity, and terminates at last in the ratio of 7 to 10 very near.

But  $x$  soon converges to the limit  $0.7q$ , so that this value of  $x$  may be assumed in all cases, let the value of  $q$  be what it will.

Some uses of this Problem will appear by the following Examples.

EXAMPLE 1.

*Let it be proposed to find in how many throws one may undertake with an equality of Chance, to throw two Aces with two Dice.*

---

*Date:* 1756.

The number of Chances upon two Dice being 36, out of which there is but one chance for two Aces, it follows that the number of Chances against it is 35; multiply therefore 35 by 0.7, and the product 24.5 will shew that the number of throws requisite to that effect will be between 24 and 25.

EXAMPLE 2.

*To find in how many throws of three Dice, one may undertake to throw three Aces.*

The number of all the Chances upon three Dice being 216, out of which there is but one Chance for 3 Aces, and 215 against it, it follows that 215 ought to be multiplied by 0.7; which being done, the product 150.5 will shew that the number of Throws requisite to that effect will be 150, or very near it.

EXAMPLE 3.

*In a Lottery whereof the number of blanks is to the number of prizes as 39 to 1, (such as was the Lottery in 1710) to find how many Tickets one must take to make it an equal Chance for one or more Prizes.*

Multiply 39 by 0.7, and the product 27.3 will shew that the number of Tickets requisite to that effect will be 27 or 28 at most.

Likewise in a Lottery whereof the number of Blanks is to the number of Prizes as 5 to 1, multiply 5 by 0.7, and the product 3.5 will shew that there is more than an equality of Chance in 4 Tickets for one or more Prizes, but less than an equality in three.