THE GAME OF BASSETTE

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Rules of the Play.

The Dealer, otherwise called the *Banker*, holds a pack of 52 Cards, and having shuffled them, he turns the whole pack at once, so as to discover the last Card; after which he lays down by couples all the Cards.

The Setter, otherwise called the *Ponte*, has 13 Cards in his hand, one of every sort, from the King to the Ace, which 13 Cards are called a *Book*; out of this Book he takes one Card or more at pleasure, upon which he lays a Stake.

The Ponte may at his choice, either lay down his Stake before the pack is turned, or immediately after it is turned; or after any number of Couples are drawn.

The first case being particular, shall be calculated by itself; but the other two being comprehended under the same Rules, we shall begin with them.

Supposing the Ponte to lay down his Stake after the Pack is turned, I call 1, 2, 3, 4, 5, &c. the places of those Cards which follow the Card in view, either immediately after the pack is turned, or after any number of couples are drawn.

If the Card upon which the Ponte has laid a Stake comes out in any odd place, except the first, he wins a Stake equal to his own.

If the Card upon which the Ponte has laid a Stake comes out in any even place, except the second, he loses his Stake.

If the Card of the Ponte comes out in the first place, he neither wins nor loses, but takes his Stake again.

If the Card of the Ponte comes out in the second place, he does not lose his whole Stake, but only a part of it, *viz.* one half, which to make the Calculation more general we shall call y. In this case the Ponte is said to be *Faced*.

When the Ponte chooses to come in after any number of Couples are down; if his Card happens to be but once in the Pack, and is the very last of all, there is an exception from the general Rule; for tho' it comes out in an odd place, which should intitle him to win a Stake equal to his own, yet he neither wins nor loses from that circumstance, but takes back his own Stake.

PROBLEM XIII.

To estimate at Bassette *the Loss of the* Ponte *under any circumstance of Cards remaining in the Stock, when he lays his Stake; and of any number of times that his Card is repeated in the Stock.*

The Solution of this Problem containing four cases, *viz.* of the Ponte's Card being *once, twice, three* or *four* times in the Stock; we shall give the Solution of all these cases severally.

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SOLUTION of the *first* Case.

The Ponte has the following chances to win or lose, according to the place his Card is in.

1	1	Chance for winning	0
2	1	Chance for losing	y
3	1	Chance for winning	1
4	1	Chance for losing	1
5	1	Chance for winning	1
6	1	Chance for losing	1
$*$	1	Chance for winning	0

It appears by this Scheme, that he has as many Chances to win 1 as to lose 1, and that there are two Chances for neither winning or losing, *viz.*, the first and the last, and therefore that his only Loss is upon account of his being *Faced:* from which it is plain that the number of Cards covered by that which is in view being called n, his Loss will be $\frac{y}{n}$, or $\frac{1}{2n}$, supposing $y = \frac{1}{2}$.

SOLUTION of the *second* Case.

By the first Remark belonging to the xth Problem, it appears[1] that the Chances which the Ponte has to win or lose are proportional to the numbers, $n-1$, $n-2$, $n-3$, &c. Wherefore his Chances for winning and losing may be expressed by the following Scheme.

1	$n-1$	Chance for winning	0
2	$n-2$	Chance for losing	y
3	$n-3$	Chance for winning	1
4	$n-4$	Chance for losing	1
5	$n-5$	Chance for winning	1
6	$n-6$	Chance for losing	1
7	$n-7$	Chance for winning	1
8	$n-8$	Chance for losing	1
9	$n-9$	Chance for winning	1
$*$	1	Chance for losing	1

Now setting aside the first and second number of Chances, it will be found that the difference between the 3rd and 4th is $= 1$, that the difference between the 5th and 6th is also $= 1$, and that the difference between the 7th and the 8th is also $= 1$, and so on. But the number of differences is $\frac{n-3}{2}$, and the Sum of all the Chances is $\frac{n}{1} \times \frac{\overline{n-1}}{2}$: wherefore the Gain of the Ponte is $\frac{n-3}{n\times\overline{n-1}}$. But his Loss upon account of the *Face* is $\overline{n-2} \times y$ divided by $\frac{n\times\overline{n-1}}{1\times 2}$ that is $\frac{\overline{2n-4}\times y}{n\times\overline{n-1}}$: hence it is to be concluded that his Loss upon the whole is $\frac{\overline{2n-4}\times y-\overline{n-3}}{n\times\overline{n-1}}$ or $\frac{1}{n\times\overline{n-1}}$ supposing $y = \frac{1}{2}$.

That the number of differences is $\frac{n-3}{2}$ will be evident from two considerations.

First, the Series $n-3$, $n-4$, $n-5$, &c. decreases in Arithmetic Progression, the difference of its terms being Unity, and the last Term also Unity, therefore the number of its Terms is equal to the first Term $n-3$: but the number of differences is one half of the number of Terms; therefore the number of differences is $\frac{n-3}{2}$.

Secondly, it appears, by the xth Problem, that the number of all the Terms including the two first is always $b+1$, but a in this case is $= 2$, therefore the number of all the Terms is

[1]Namely, by calling the Ponte's two Cards two white Counters, drawn for alternately by A and B; and supposing all A's Chances to belong to the Banker's right hand, and those of B to his left. And the like for the Cases of the Ponte's Card being in the Stock 3 or 4 times.

$n-1$; from which excluding the two first, the number of remaining Terms will be $n-3$, and consequently the number of differences $\frac{n-3}{2}$.

That the Sum of all the Terms is $\frac{n}{1} \times \frac{n-1}{2}$, is evident also from two different considerations.

First in any Arithmetic Progression whereof the first Term is $n-1$, the difference Unity, and the last Term also Unity, the Sum of the Progression will be $\frac{n}{1} \times \frac{n-1}{2}$.

Secondly, the Series $\frac{2}{n \times n-1} \times \overline{\overline{n-1} + \overline{n-2} + \overline{n-3}}$, &c. mentioned in the first Remark upon the tenth Problem, expresses the Sum of the Probabilities of winning which belong to the several Gamesters in the case of two white Counters, when the number of all the Counters is n. It therefore expresses likewise the Sum of the Probabilities of winning which belong to the Ponte and Banker in the present case: but this Sum must always be equal to Unity, it being a certainty that the Ponte or Banker must win; supposing therefore that $n-1+n-2+n-3$, &c. is $= S$, we shall have the Equation $\frac{2S}{n \times n-1} = 1$, and therefore $S = \frac{n}{1} \times \frac{n-1}{2}$.

SOLUTION of the *third* Case.

By the first Remark of the tenth Problem, it appears that the Chances which the Ponte has to win and lose, may be expressed by the following Scheme.

1	$n-1 \times n-2$	Chance for winning	0
2	$n-2 \times n-3$	Chance for losing	y
3	$n-3 \times n-4$	Chance for winning	1
4	$n-4 \times n-5$	Chance for losing	1
5	$n-5 \times n-6$	Chance for winning	1
6	$n-6 \times n-7$	Chance for losing	1
7	$n-7 \times n-8$	Chance for winning	1
8	$n-8 \times n-9$	Chance for losing	1
$*$	2×1	Chance for winning	1

Setting aside the first, second, and last number of Chances, it will be found that the difference between the 3rd and 4th is $2n-8$; the difference between the 5th and 6th, $2n-12$; the difference between the 7th and 8th, $2n-16$, &c. Now these differences constitute an Arithmetic Progression, whereof the first Term is $2n-8$, the common difference 4, and the last Term 6, being the difference between 4×3 and 3×2. Wherefore the Sum of this Progression is $\frac{n-1}{1} \times \frac{n-5}{2}$, to which adding the last Term 2×1, which is favourable to the Ponte, the Sum total will be $\frac{n-3}{1} \times \frac{n-3}{2}$: but the Sum of all the Chances is $\frac{n}{1} \times \frac{n-1}{1} \times \frac{n-2}{3}$, as may be collected from the first Remark of the xth Problem, and the last Paragraph of the second case of this Problem: therefore the Gain of the Ponte is $\frac{3.n-3.n-3}{2.n.n-1.n-2}$. But his Loss upon account of the Face is $\frac{3.n-2.n-3.y}{n.n-1.n-2}$ or $\frac{3y.n-3}{n.n-1}$, therefore his Loss upon the whole is $\frac{3y.n-3}{n.n-1} - \frac{3.n-3.n-3}{2.n.n-1.n-2}$; or $\frac{3n-9}{2.n.n-1.n-2}$ supposing $y = \frac{1}{2}$.

SOLUTION of the *fourth* Case.

The Chances of the Ponte may be expressed by the following Scheme.

1	$n-1\times n-2\times n-3$	for winning	0
2	$n-2\times n-3\times n-4$	for losing	y
3	$n-3\times n-4\times n-5$	for winning	1
4	$n-4\times n-5\times n-6$	for losing	1
5	$n-5\times n-6\times n-7$	for winning	1
6	$n-6\times n-7\times n-8$	for losing	1
7	$n-7\times n-8\times n-9$	for winning	1
$*$	$3\times 2\times 1$	for winning	1

Setting aside the first and second numbers of Chances, and taking the differences between the 3rd and 4th, 5th and 6th, 7th and 8th, the last of these differences will be found to be 18. Now if the number of those differences be p, and we begin from the last 18, their Sum, from the second Remark of the xth Problem, will be found to be $p\times\overline{p+1}\times\overline{4p+5}$, but p in this case is $=\frac{n-5}{2}$, and therefore the Sum of these differences will easily appear to be $\frac{n-5}{2}\times\frac{n-3}{2}\times\frac{2n-5}{1}$, but the Sum of all the Chances is $\frac{n}{1}\times\frac{n-1}{1}\times\frac{n-2}{1}\times\frac{n-3}{4}$; wherefore the Gain of the Ponte is $\frac{n-5.n-3.2n-5}{n.n-1.n-2.n-3}$: now his Loss upon account of the Face is $\frac{n-2.n-3.n-4.4y}{n.n-1.n-2.n-3}$, and therefore his Loss upon the whole will be $\frac{\overline{n-4}.4y}{n.n-1}-\frac{n-5.2n-5}{n.n-1.n-2}$ or $\frac{3n-9}{n.n-1.n-2}$, supposing $y=\frac{1}{2}$.

There still remains one single case to be considered, *viz.* what the Loss of the Ponte is, when he lays a Stake before the Pack is turned up: but there will be no difficulty in it, after what we have said; the difference between this case and the rest being only, that he is liable to be faced by the first Card discovered, which will make his Loss to be $\frac{3n-6}{n.n-1.n-3}$, that is, interpreting n by the number of all the Cards in the Pack, *viz.* 52, about $\frac{1}{866}$ part of his Stake.

From what has been said, a Table may easily be composed, shewing the several Losses of the Ponte in whatever circumstance he may happen to be.

The use of this Table will be best explained by some Examples.

EXAMPLE 1.

Let it be proposed to find the Loss of the Ponte, when there are 26 Cards remaining in the Stock, and his Card is twice in it.

In the Column N find the number 25, which is less by 1 than the number of Cards remaining in the Stock: over-against it , and under the number 2, which is at the head of the second Column, you will find 600; which is the Denominator of a fraction whose Numerator is Unity, and which shews that his Loss in that circumstance is one part in six hundred of his Stake.

EXAMPLE 2.

To find the Loss of the Ponte when there are eight Cards remaining in the Stock, and his Card is three times in it.

In the Column N find the number 7, less by one than the number of Cards remaining in the Stock: over-against 7, and under the number 3, written on the top of one of the Columns, you will find 35, which denotes his Loss is one part in thirty-five of his Stake.

COROLLARY 1.

'Tis plain from the construction of the Table, that the fewer Cards are in the Stock, the greater is the Loss of the Ponte.

N	1	2	3	4
52	***	***	***	866
51	***	***	1735	867
49	98	2352	1602	801
47	94	2162	1474	737
45	90	1980	1351	675
43	86	1806	1234	617
41	82	1640	1122	561
39	78	1482	1015	507
37	74	1332	914	457
35	70	1190	818	409
33	66	1056	727	363
31	62	930	642	321
29	58	812	562	281
27	54	702	487	243
25	50	600	418	209
23	46	506	354	177
21	42	420	295	147
19	38	342	242	121
17	34	272	194	97
15	30	210	151	75
13	26	156	114	57
11	22	110	82	41
9	18	72	56	28
7	14	42	35	17

COROLLARY 2.

The least Loss of the Ponte, under the same circumstances of Cards remaining in the Stock, is when his Card is but twice in it; the next greater but three times; still greater when four times; and the greatest when but once. If the Loss upon the Face were varied, 'tis plain that in all the like circumstances, the Loss of the Ponte would vary accordingly; but it would be easy to compose other Tables to answer that variation; since the quantity y, which has been assumed to represent that Loss, having been preserved in the general expression of the Losses, if it be interpreted by $\frac{2}{3}$ instead of $\frac{1}{2}$, the Loss, in that case, would be as easily determined as in the other: thus supposing that 8 Cards are remaining in the Stock, and that the Card of the Ponte is twice in it, and also that y should be interpreted by $\frac{2}{3}$, the Loss of the Ponte would be found to be $\frac{4}{63}$ instead of $\frac{1}{42}$.