

Annotatio de quibusdam Ludis; imprimis de Ludo quodam Sinico, differentiaque Scachici & Latrundulorum, & novo genere Ludi Navalis.

A Note on certain Games, especially a certain Chinese Game, and on the Difference between Chess and Latrunculus and on a new kind of Naval Game.

Gottfried Wilhelm Leibniz¹
Miscellanea Berolinensia, 1710, pp. 22–26

Saepe notavimus, nusquam homines quam in ludicris ingeniosiores esse: atque ideo ludos Mathematicorum curam mereri, non per se, sed artis inveniendi causa. Ludi eventus fortuiti inter alia prosunt ad aestimandas probabilitates; habemusque ingeniosissimas de alea ratiocinationes, quibus occasionem dedit Fques Meraeus (*le Chevalier de Meré*) notus in vulgus libello, quomodo jucundi esse possimus (*des agréments*) qui Blasio Pascasio primum Quaestiones proposuit, quas & his & Christianus Huygenius, & Frane Fermatius, summi Mathematici pulcherrimè solverunt. Unde nata sunt resolvendarum diversissimarum longaque majoris momenti Quaestionum Fundamenta. Inde autem factum est, ut Meraeus vir ingeniosus, sed semidoctus, &, ut ita dicam semi-scius, cum sola vi ingenii praevidisset, quae postea tanti viri mathematicae certitudinis habitu induerunt, successu laudibusque tumens, Doctoris personam sibi sumeret in Pascaliū, nescio qua jam tum remissione animi inter Mathematica devotionemque praeposteram fluctuantem; ut apparet ex mirabili illius ad hunc Epistola (quae est decima nona in Miscellaneis Meraei) ex qua Baylius quaedam refert in dictionario Critico, artic. de Zenone; non satis intelligens, aut quid hos animos dederit Meraeo, ut sua, inventa jactaret, dum Geometrarum demonstrationes irridet, falsasque proclamat,

We have often remarked, on no occasion are men more clever than in their diversions: and for that reason games be merited the attention of Mathematicians, not for themselves, but for the sake of discovering knowledge. Games of chance outcome among others are useful to estimate probabilities; and we have the most clever reasonings on chance, on which the famous Fques Meraeus (*the Chevalier de Meré*) has given to the public in a small work, in what way we are able to be agreeable (*des agréments*) who first has proposed Questions to Blaise Pascal, which both this one & Christian Huygens, & the Frenchman Fermat, most eminent Mathematicians, have solved most beautifully. Whence have been born the Fundamentals of resolving the most diverse Questions and of by far greater merit. But thence it has happened, that the ingenious, but half-taught, man Méré, &, as thus I say semi-skilled, by strength of talent alone, had foreseen, what afterward such great men have clothed in the garment of mathematical certitude, swelling with success and with renown, he himself assumed the character of teacher toward Pascal, I know not now at that time by what relaxation of the mind wavering between Mathematics and wrong-headed piety; that it is evident from an extraordinary letter of that man on this (which is nineteen of the *Mélanges* of Méré) out of which a certain Bayle re-

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& continui divisibilitatem in infinitum (pervulgatis ferè sophismatibus) impugnat; aut quid omnino sibi voluerit homo alias parum philosophus, dum ad Mundum quendam invisibilem provocat, in quo rationes rerum principiaque veritatesque arcanæ, omnisque convenientiæ & certitudinis fontes laterent. Sanè verissimum est, esse superiorem Mathematica scientiam, parem certitudine majorem virtute atque efficacia, ubi rationes ideales, non tantùm à sensibus, sed etiam ab imaginibus sejunguntur; & hujus rei aliquid interviderat Meraeus: sed addictior erat sensibili Mundo, quam ut satis id illum intelligibilem admitti posset.

Ludi Misti ex fortuna & ingenio apertissime vitam humanam, sed maxime res militares & praxin medicam repræsentant; ubi pars arti, pars casui necessario danda est; ut in ludo, qui à versa vice (*verkehren*) apud Germanos nomen invenit, cui vicinus *Tric-trac*. Et hic conjungendæ sunt consequentiæ certæ cum verisimilitudinum aestimatione.

Sed maximè scientiæ substant Ludi, qui unicè arti eventum, nihil casui debent: in quibus haud-dubiè eminet *Ludus Scachicus* seu Regius, ubi duo Reges, legatis, tribunis, equitibus, peditibusque stipati acie confligunt; de quo justum opus sanè ingeniosissimum olim Germanico Sermone edidit Gustavus Selenus, ist est, Augustus Luneburgicus postea Dux Welfebytanus, princeps multis modis laudem meritis.

ports in the *Dictionnaire Critique*, the article on Zenon; not adequately understanding, or at least what may have given these thoughts to Méré, as he boasted his discoveries, while he ridicules and proclaims false the demonstrations of Geometers, & he attacks the division of the continuum to infinity (by spreading about sophisms generally); or rather what the philosophical man scarcely elsewhere brings about entirely to himself, while he appeals to a certain invisible World with regard to which reasons for things and beginnings and arcane truths, and all consistencies & principle causes of certitude must lie hidden. Certainly it is most true, a science to be superior to Mathematics, equal in certitude greater in power and in effectiveness, where ideal ratios, are separated not so much from sense, but also from images; & Méré had seen this thing to some degree: but he was more devoted to the sensible World, than rather that it was able to be admitted into that understandable one.

The Games Mixed out of chance & talent most closely represent human life, but chiefly military things & medical practice; where the part to art, the part to cause is surrendered necessarily; as in the case of the game, which finds the name from versa vice (*verkehren*)¹ among the Germans, to that similar *Tric-trac*. And here certain consequences are associated with the estimation of verisimilitude.

But chiefly Games stand firm to science, which especially the event to art, owe nothing to chance: in which without doubt *The Chess Game* or Royal stand out, where two Kings, surrounded with legates, tribunes, knights, and infantrymen collide in battle line; about which Gustavus Selenus, that is, Auguste Luneburg later Duke of Wolfenbüttel, a prince in many ways merited praise, has published in German Language, a just work

Facile crediderim, Graecos & Romanos veteres habuisse Ludum quendam Regium, sed non apparet, similem fuisse *Scachico*, qui ex Oriente fortasse seriùs venit. Saltem illud pro certo habeo (quamquam aliter cum plerisque sentiat summae Vir doctrinae Claudius Salmasius) Scachicum à *ludo Latrunculorum* toto caelo abfuisse, quod ex lege hujus primaria probo, quam nobis Poëtae conseruarunt. Sic enim Ovidius de Arte Amandi lib. 3.

Cautaque non stultè latronum proelia ludat,
Unus cum gemino miles ab hoste perit.²

Et Martial lib. 14 Epigr. 17

Calculus hic gemino discolor hoste perit.³

Ea lex valde rationi consentanea erat, ut plus valerent duo milites quam unus; atque adeo unus calculus inter duos hostiles velut per insidias delapsus amitteretur. Sed hujus legis in Ludo Scachico nostro nullum est vestigium, neque natura ejus illam capit.

Non ita pridem increbuit ludi genus singulare, quem *Solitarium* appellant, ubi ego ipse solus ludo, sed collusore velut teste & arbitro, an faciam officio satis. Plena est area calculis in foramina infixis, qui ordine eximendi sunt, sed nullus, demto primo, qui pro arbitrio eligitur eximi potest, nisi alium calculum transsiliendo devenire possis proxime in vacuum locum, ut in ludo Damico caeditur. Qui ea lege servata omnes ad novissimum usque eximit, vincit; Sed qui uno plures tandem relinquere in area cogitur,

indeed often most ingenious.

Willingly I have believed the ancient Greeks & Romans to have had a certain Royal Game, but it is not apparent, to have been similar to *Chess*, which possibly came late out of the Orient. At least for certain I have the former (nevertheless with the majority the man Claude Saumaise of greatest learning perceives otherwise) *Chess* to have been distinct from the Game of *Latrunculus* to the whole world, because I prove out of the first principle of this, as the Poets have preserved for us. For namely Ovid in *Ars Amatoria*, book 3.

And Martial, book 14, Epigram 17.

That law was greatly conformed to reason, that two soldiers are stronger than one; and to such a degree a single pebble between two enemies is lost just as if fallen by ambush. But there is no trace of this principle to us in the *Chess Game*, and the nature of it does not capture that.

Not particularly long ago a singular kind of game has become common, which they call *Solitaire*, which I myself play alone, but with a companion just as witness & judge, or that I play with dutiful action. The board is an area with pebbles thrust in depressions, which are extracted in order, but none, with the first excepted, which is chosen arbitrarily, is able to be released, unless you may be able to reach another pebble by passing over the nearest into empty space, as it is slaughtered in the *Game of Draughts*.² In which way

²She should play the fights of robbers with caution not foolishly, where one soldier is lost to two from the enemy.

³The pebble of different color is taken by twin enemy.

palma excidit. Elegantiùs inverti potest hic ludus, post unum in area vacua pro arbitrio positum, caeteros collacando, sed ea lege observata ponendi, quae fuit paulo ante dicta eximendi. Ita possumus vel aream implere, vel, quod foret ingeniosius, datam figuram calculis formare; velut trigonum, quadratum, octogonum, aut aliud quiddam, si modo licet; neque enim res semper possibilis foret: atque hoc ipsum magnae artis esset praevidere, quid praestari posset; haberetque aliquid imprimis Geometricum hic processus. Sed ego ad profectum inventricis artis ludendi artificia detexisse, non ludum valde exercuisse laudarem.

Venio ad expositionem Ludi Sinici, cuius occasione caetera afferre placuit. Habetur ejus repraesentatio in libro Figurarum Sinicarum qui extat in Bibliotheca Inclyti Regis Borussorum Berolinensi, unde eam aere exprimi curavimus [Fig. I]. Ludus est ex eorum genere, qui sola arte constant, nulla mistura fortunae: caeterum id habet singulare, quod collusores (quantum apparet) calculos sibi invicem non tollunt, sed tantum sese obsident atque coarctant; ut vincat tandem, sed, ut sic dicam, sine caede & sanquine, qui alteri adimit movendi se libertatem. Quod sane in aliis ludis non raro, hic autem necessario contingit.

Sed audiamus hoc de ludo Nicolaum Triautium capite octavo primi librorum de Christiana expeditione apud Sinas, ex Matthaei Riccii insignis Viri, & veri

he served by the principle removes all to the last continuously, wins; but in which way first place disappears with one who is forced finally to abandon many in an area. More elegantly this game is able to be inverted, after one is placed in an empty area arbitrarily, by arranging the rest, but they to be placed observed by the rule, which has been aforesaid just above, must be removed. Thus we are able either to fill the area, or, because it was more clever, to form a given figure with pebbles; as a triangle, square, octagon, or something other, if only it is permitted: and not for instance a thing always was possible: and this itself is by great art to foresee, what was able to be furnished; and this process should have in the first place a geometrical character. But I should call to the accomplished of the art of invention of playing to have revealed skills, not to have practiced a game vigorously.

I come to the exposition of the Chinese Game, of which by chance it is pleased to bring the rest. A representation of it is had in the book of Chinese Figures which exists in the Library of the Celebrated King of Prussia in Berlin, whence we have arranged it to be expressed in bronze [Fig. I]. The game is out of the kind of them which is dependent alone on art, not on the mixing of chance: otherwise it has alone, because the gamblers (it is apparent how many) do not remove pebbles from one another by turns, but will occupy and restrict so much with themselves; that finally he must win, but, as thus I said, without slaughter and bloodshed, who withdraws freedom itself of being moved from the latter. Because certainly in other games not seldom, moreover this happens necessarily.

But let us hear Nicolas Trigaut concerning this game in the eighth chapter of the first of the books concerning the Christian expedition among the Chi-

Sinensium missionum fundatoris, commentariis confectorum. Sic ergo loquitur: *Gravissimum inter eos ludi genus est hujus modi: in alveo, trecentarum Cellularum plures* [hanc vocem tollendam puto vel legendum *plus quam trecentarum cellularum*, fortasse vitio Latini interpretis Riccii Italica an Trigautii Gallica, male exprimentis; neque enim collusores sunt nisi duo] *ducentis calculis colludunt, ex quibus alii candidi sunt, alii atrii. His calculis alter alterius calculos procurat in medium alveum relegare, ut reliquis deinde cellulis dominetur; ad postremum, qui plures in alveo cellulas sibi subiecit, victor appellatur. Hunc ludum avidissime arripiunt Magistratus, et saepe maximam diei partem ludendo consumunt; nam inter ludendi peritos horam integram ludus unus tenet. Qui hujus ludi peritus est, tam etsi nulla aliare insignis fuerit, ab omnibus colitur et evocatur. Imo nonnulli etiam eos solitis sibi ritibus Magistros legunt, ut ab illis accuratè hujus ludi rationem ediscant. Haec Trigautius.*

Sed in hac descriptione peccatum ocularis inspectio schematis ostendit. Est enim Alveus figurae quadratae, cujus unum latus est *octodecim cellularum*: unde cellulae in universum sunt octies & decies octodecim, id est, non trecentae, sed *trecentae viginti quatuor*. Et parum ad rem facit descriptio reliqua: nam ut unus alterum in medium alvei semper contradat, manifestum est, in potestate semper, atque adeo necessarium non esse; cum sufficiat obsessum inclusisse, sive in medio sive in angulo. Neque is vincit, qui pluribus cellis dominatur [quæ enim hoc definitas] nisi cum hoste clauso solus cam-

nese, constructed from the commentaries of Mattheus Ricci famous man & true founder of Chinese missions. Thus therefore he speaks: *The most serious among them is the kind to be played in this manner: in a gaming table of three hundred cells many* [I think this expression must be removed or must be read *more than three hundred cells*, expressing badly perhaps by defect of the Latin translator from the Italian of Ricci or from the French of Trigaut; for the players are but two] *play with two hundred pebbles, out of which some are white, others black. With these pebbles the one manages the pebbles of the other to be relegated to the middle the table, in order that next he may dominate over the cells with the rest; finally, who has subjected more cells to himself on the table, is called winner. Magistrates take up this game most avidly, and often consume the greatest part of the day by playing; for between the skilled at play one game holds an entire hour. Who of this game is expert, although to such an extent was not distinguished to another, is honored and called forth by all men. Indeed some yet gather Magistrates to themselves by customary ceremonies, in order that by those they study the rules of this game carefully. This Trigaut.*

But in this description of the figure observation by the eye has revealed the mistake. For the Gameboard is of square figure, of which one side is *eighteen cells*: whence of the cells in the whole there are eight & ten times eighteen, that is, not three hundred, but *three hundred twenty-four*. And the description leaves a remainder insufficient to this: for as one may crowd another in the middle of the game board always, it is manifest, in opportunity always, and truly not to be necessary, when it is sufficient to have the besieged either in the middle or at an angle. And he conquers not, who dominates with many

pum liberum tenet.

Facit multitudo calculorum & magnitudo alvei, ut facile credam, ludum hunc plurimum habere ingenii & difficultatis, tametsi omnes ejus leges nobis non sint notae. Certe singularis ejus ratio, ut scilicet omnia non caedendo hostem sed tantum ad incitas redigendo transigantur (quod non est in nostris ludis) memorari merebatur. Et credibile est, Brachmanem aliquem fuisse inventorem, qui à caede abhorrens incruentam victoriam volebat. Nam constat, plerosque Indiae Orientalis populos, Christianiores, ut sic dicam, ea in re, his, qui Christiani vocantur, caedes in ipso bello refugere solere.

Venit etiam in mentem comminisci novum Ludi genus, quem *Ludum Navalem* appelles, ubi velut classibus confligatur in Tabula tanquam in imaginario mari, calculis naves exhibentibus determinata venti plaga, unde & obliquitas viae contra ventum maxima, & pro angulo motus velocitas praescribatur; qua ratione *Evolutiones Navales* quam proximè repraesentarentur, & ludendo exercerentur.

- 1· It seems that the game Verkehren is similar to Backgammon or Trictrac.
- 2· Jeu des Dames, or a variant of Checkers.

cells [for which you must define this] except the one holds with enemy enclosed a free open space.

The multitude of pebbles & size of the gameboard makes, as I should believe easily, this game to have the greatest cleverness and difficulty, although all rules of it may not be known to us. Certainly the singular reckoning of it, that namely all finish not by slaughtering the enemy but only by reducing the unmoved ones (which is not in our games) was merited to be remembered. And it is credible, some Brahman to be the inventor, who by abhorring slaughter wished bloodless victory. For it is evident, most people of the Orient of India, more Christian, so I say thus, it in the thing, than those, who are called Christian, to be accustomed to flee slaughter in war itself.

In addition there comes to mind a new kind of game I have devised, which you may call *Naval Game*, where he may contend as if with fleets on a Table just as if on an imaginary sea, with pebbles presenting ships with the gust of the wind determined, whence also the maximum obliquity of direction against the wind, & for angle the velocity of movement may be prescribed; by which reason *Naval Developments* may be represented as nearly as possible, & may be exercised by playing.