APPLICATION DU CALCUL DES PROBABILITÉS

AUX OPÉRATIONS GÉODÉSIQUES DE LA MÉRIDIENNE

Pierre Simon Laplace*

Connaissance des Temps for the year 1822 (1820) pp. 346–348.

The part of the meridian, which extends from Perpignan to Formentera, rests on the base measured near Perpignan. Its length is around 460 thousand meters, and it is joined to the base by a chain of twenty-six triangles. We can fear that such a great length which has not been verified at all by the measure of a second base toward its other extremity, is susceptible of a sensible error arising from the errors of the twenty-six triangles employed in measuring it. It is therefore interesting to determine the probability that this error not exceed forty or fifty meters. Mr. Damoiseau, leutenant-colonel of the artillery, who has just gained the prize proposed by the Academy of Turin, on the return of the comet of 1759, has well wished, at my request, to apply to this part of the meridian, the formulas that I have given for this object, in the second Supplement to my Théorie analytique des Probabilités. He has found that by departing from the latitude of the signal of Burgarach, some minutes more to the north than Perpignan, to Formentera, that which comprehends an arc of the meridian of around 466006 meters, the probability of an error s, is proportional to the exponential

$$c^{\frac{-9ns^2}{4\theta^2.48350,606}},$$

c is the number of which the logarithm is unity; n is the number of triangles employed, θ^2 is the sum of the squares of the errors observed in the sum of the three angles of each triangle; finally s is the error of the total arc, the base of Perpignan being taken for unity. Here n is equal to 26. By taking for unity of angle, thed sexagesimal secon, we have

$$\theta^2 = 118,178.$$

But the number of triangles employed being only 26, it is preferable to determine by a great number of triangles, the constant θ^2 which depends on the unknown law of the errors of the partial observations. For that, we have made use of the one hundred seven triangles which have served to measure the meridian from Dunkirk, to Formentera. The collection of the errors of the observed sums of the three angles of each triangle is, by

*Translated by Richard J. Pulskamp, Department of Mathematics & Computer Science, Xavier University, Cincinnati, OH. June 20, 2012 taking them all positively, 173,82: the sum of the squares of these errors, is 445,217. By multiplying it by $\frac{26}{107}$, we will have for the value of θ^2

$$\theta^2 = 108,134.$$

This value which differs little from the preceding, must be employed in preference. It is necessary to reduce it into parts of the radius of the circle, by dividing it by the square of the number of sexagesimal seconds that this radius contains; then the preceding exponential becomes

$$c^{-(689,797)^2.s^2};$$

so that the base of Perpignan being taken for unity, $(689,797)^2$ is that which I name *the weight* of the result or of the arc measured from the signal of Burgarach, to Formentera. This base is of 11706,40 m; we have concluded from it for the respective probabilities that the errors of the arc of which there is concern, are comprehended within the limits $\pm$60 m, $\pm$50 m, $\pm$40 m, the following fractions which approach quite nearly unity,

$$\frac{1743695}{1743696}, \quad \frac{32345}{32346}, \quad \frac{1164}{1165}.$$

We must therefore have no reasonable doubt on the exactitude of the measured arc. The limits between which there are odds one against one, that the error falls, are $\pm 8,0937$ m.

If we measured on the side of Spain, a base for verification, equal to the base of Perpignan, and if we joined it by two triangles, to the chain of the triangles of the meridian; we find by the calculation, that we can wager one against one, that the difference between the measure of this base, and its value concluded from the base of Perpignan, would not surpass a third of a meter: this is nearly the difference in the measure of the base of Perpignan, to its value concluded from the base of Melun.

We have seen in the Supplement cited, that the angles having been measured by means of a repeating circle; we are able to suppose the probability of an error x in the sum observed of the three angles of each triangle, proportional to the exponential c^{-kx^2}, k being a constant; whence it follows that the probability of this error is

$$\frac{dx.\sqrt{k}.c^{-kx^2}}{\sqrt{\pi}},$$

π designating the ratio of the circumference to the diameter.

By multiplying it by x, taking the integral from x null to x infinity, and doubling this integral; we will have clearly the mean error, by taking positively the negative errors. This mean error being therefore designated by ϵ, we will have

$$\epsilon = \frac{1}{\sqrt{k\pi}}.$$

We will have the mean value of the squares of these errors, by multiplying by x^2 the preceding differential, and integrating from $x = -\infty$, to x infinity; by naming therefore ϵ' this value, we will have

$$\epsilon' = \frac{1}{2k}.$$

Thence, we deduce

$$\epsilon' = \frac{\epsilon^2.\pi}{2}.$$

We can thus obtain θ^2, by means of the errors taken all to plus, of the sum observed of the angles of each triangle. In the one hundred seven triangles of the meridian, this sum is by that which precedes, 172,82; we can thus take for ϵ, $\frac{173{,}82}{107}$; that which gives $26.\epsilon'$, or for θ^2

$$\theta^2 = \frac{26\pi}{2} \cdot \left(\frac{173,82}{107}\right)^2 = 107,78.$$

This differs very little from the value 108,134 given by the sum of the squares of the errors of the sum observed of the angles of each of the one hundred seven triangles. This accord is remarkable.

By supposing the angle of intersection of the base of Perpignan, with the meridian which passes through one of the extremities of this base, well determined; we would have exactly the angle of intersection of the meridian with the last side of the chain of the triangles which unite this base to the isle of Formentera, if the earth were a spheroid of revolution, and if the angles of the triangle were exactly measured. The error coming from this second cause, in the last angle of intersection, is by the formulas of the second supplement cited, proportional to the exponential c^{-r^2}, by expressing this error by $\frac{2}{3}\theta r$ which in the present case becomes $6'', 8997.r$. Thence it follows that the limits between which we can wager one against one, that the error falls, are $\pm 3'', 2908$. If the azimuthal observations were made with a very great precision; we would determine by this formula, the probability that they indicate an ellipticity in the terrestrial parallels.

We can estimate the relative exactitude of the instruments of which we make use in the geodesic operations, by the value of ϵ' concluded from a great number of triangles. That value concluded from one hundred seven triangles of the meridian, is $\frac{445.217}{107}$. The same value concluded from forty-three triangles employed by La Condamine, in his measure of the three degrees of the equator, is $\frac{1718}{43}$, or near ten times greater than the preceding. The equally probable errors, relative to the instruments employed in these two operations, are proportionals to the square roots of the values of ϵ'. Thence it follows that the limits $\pm 8, 0937$ m, between which we just saw that it is equally probable that the error of the measured arc from Perpignan to Formentera falls, would have been $\pm 25, 022$ m with the instruments employed by La Condamine. These limits would have surpassed ± 40 m, with the instruments employed by LaCaille and Cassini, in their measure of the meridian. We see thus how much the introduction of the repeating circle in the geodesic operations, has been advantageous.