BOOK II
CHAPTER X
DE L'ESPÉRANCE MORALE.

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ON MORAL EXPECTATION

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§41. We have seen in §2, the difference which exists between mathematical expectation and moral expectation. Mathematical expectation resulting from the probable awaiting of one or many goods, being the product of these goods, by the probability to obtain them, it can be evaluated by the analysis exposed in that which precedes. Moral expectation is ruled on a thousand circumstances which it is nearly impossible to evaluate well. But we have given in the section cited, a principle, which being applied to the most common cases, leads to some often useful results, and of which we are going to develop the principals. [432]

According to this principle, x being the physical fortune of an individual, the increase dx that he receives, produces in the individual a moral good reciprocal to this fortune; the increase of his moral fortune can therefore be expressed by $\frac{k\,dx}{x}$, k being a constant. Thus by designating by y the moral fortune corresponding to the physical fortune x, we will have

$$y = k \log x + \log h,$$

h being an arbitrary constant that we will determine by means of a value of y corresponding to a given value of x. With respect to that, we will observe that we can never suppose x and y nulls or negatives, in the natural order of things; because a man who possesses nothing regards his existence, as a moral good which can be compared to the advantage that a physical fortune of which it is quite difficult to assign the value would procure to him, but that we can not fix below that which it would be for him rigorously necessary in order to [433]
exist; because we imagine that he would not agree at all to receive a moderate sum, such as one hundred francs, with the condition to claim nothing, when he would have spent it.

Let us suppose now that the physical fortune of an individual is a, and that the expectancy of one of the increases α, β, γ, etc., occurs to him, these quantities being able to be nulls or even negatives, that which changes the increases to diminutions. Let us represent by p, q, r, etc., the respective probabilities of these increases, the sum of these probabilities being supposed equal to unity. The corresponding moral fortunes of the individual, will be able to be

$$k\log(a+\alpha) + \log h, \quad k\log(a+\beta) + \log h, \quad k\log(a+\gamma) + \log h, \quad \text{etc.}$$

By multiplying these fortunes respectively by their probabilities p, q, r, etc.; the sum of their products will be the moral fortune of the individual, by virtue of his expectancy; by naming therefore Y this fortune, we will have

$$Y = kp\log(a+\alpha) + kq\log(a+\beta) + kr\log(a+\gamma) + \text{etc.} + \log h.$$

Let X be the physical fortune which corresponds to this moral fortune, we will have

$$Y = k\log X + \log h.$$

The comparison of these two values of Y gives

$$X = (a+\alpha)^p(a+\beta)^q(a+\gamma)^r.\text{etc.}$$

If we subtract the original fortune a, from this value from X; the difference will be the increase of the physical fortune which would procure to the individual, the same moral advantage which results for him, from his expectancy. This difference is therefore the expression of this advantage, instead that the mathematical advantage has for expression

$$p\alpha + q\beta + r\gamma + \text{etc.}$$

Thence result many important consequences. One of them is that the mathematically most equal game, is always disadvantageous. In fact, if we designate by a the physical fortune of the player before commencing the game; by p, his probability to win, and by μ his stake; [434]
that of his adversary must be, for equality of the game, $\frac{(1-p)\mu}{p}$; thus the player winning the game, his physical fortune becomes $a+\frac{1-p}{p}\mu$, and the probability of that is p. If he loses the game, his physical fortune becomes $a-\mu$, and the probability of that is $1-p$; by naming therefore X his physical fortune, by virtue of his expectation, we will have by that which precedes,

$$X = \left(a + \frac{1-p}{p}\mu\right)^p (a-\mu)^{1-p};$$

now this quantity is smaller than a, that is that we have

$$\left(1 + \frac{1-p}{p}\cdot\frac{\mu}{a}\right)^p \left(1-\frac{\mu}{a}\right)^{1-p} < 1$$

or by taking the hyperbolic logarithms,

$$p\log\left(1 + \frac{1-p}{p}\cdot\frac{\mu}{a}\right) + (1-p)\log\left(1-\frac{\mu}{a}\right) < 0.$$

The first member of this equation can be put under the form

$$\int (1-p)\cdot\frac{d\mu}{a}\left(\frac{1}{1+\frac{1-p}{p}\cdot\frac{\mu}{a}} - \frac{1}{1-\frac{\mu}{a}}\right),$$

a quantity which is evidently negative.

There results further from the preceding analysis, that it is worth more to expose his fortune, by parts, to some dangers independent from one another, than to expose all entire to the same danger. In order to show it, let us suppose that one merchant having to make come by sea, a sum ϵ, exposes it on a single vessel, and that observation has made known the probability p of the arrival of a vessel of the same kind, in the port; the mathematical advantage of the merchant, resulting from his expectation, will be $p\epsilon$. But if we represent by unity his physical fortune, independently of his expectancy; his moral fortune will be by that which precedes,

$$kp\log(1+\epsilon) + \log h,$$

and his moral advantage will be, by virtue of his expectancy, [435]

$$(1+\epsilon)^p - 1,$$

a quantity smaller than $p\epsilon$: because we have

$$(1+\epsilon)^p < 1 + p\epsilon,$$

since $\log(1+\epsilon)^p$ or $p\log(1+\epsilon)$ is less than $\log(1+p\epsilon)$, that which is evident, when we put these two logarithms under the form $\int \frac{p\,d\epsilon}{1+\epsilon}$ and $\int \frac{p\,d\epsilon}{1+p\epsilon}$.

Let us suppose now, that the merchant exposes the sum ϵ by equal parts, on r vessels. His physical fortune will become $1+\epsilon$, if all the vessels arrive, and the probability of this event is p^r. If $r-1$ vessels arrive, the physical fortune of the merchant becomes $1+\frac{(r-1)\epsilon}{r}$, and the probability of this event is $rp^{r-1}(1-p)$. If $r-2$ vessels arrive, the physical fortune of the merchant becomes $1+\frac{r-2}{r}\epsilon$, and the probability of this event is $\frac{r.\overline{r-1}}{2}p^{r-2}(1-p)^2$, and so forth; the moral fortune of the merchant is therefore by that which precedes,

$$k\left\{\begin{array}{l} p^r\log(1+\epsilon) + rp^{r-1}(1-p)\log\left(1+\dfrac{r-1}{r}\epsilon\right) \\ \quad + \dfrac{r.\overline{r-1}}{2}p^{r-2}(1-p)^2\log\left(1+\dfrac{r-2}{r}\epsilon\right) + \text{etc.}\end{array}\right\} + \log h,$$

an expression that we are able to put under this form,

$$kp\int d\epsilon\left[\frac{p^{r-1}}{1+\epsilon} + \frac{\overline{r-1}.p^{r-2}(1-p)}{1+\frac{r-1}{r}\epsilon} + \frac{\overline{r-1}.\overline{r-2}.p^{r-3}(1-p)^2}{1.2.\left(1+\frac{r-2}{r}\epsilon\right)} + \text{etc.}\right] + \log h. \qquad (a)$$

If we subtract from this expression, that of the moral fortune of the merchant, when he exposes the sum ϵ on a single vessel, and if we obtain by making $r=1$ in the preceding, that which, setting aside $\log h$, reduces that here to $kp\int\frac{d\epsilon}{1+\epsilon}$, which is equal to

$$kp\int d\epsilon\left\{\frac{p^{r-1}}{1+\epsilon} + \frac{\overline{r-1}.p^{r-2}(1-p)}{1+\epsilon} + \frac{\overline{r-1}.\overline{r-2}.p^{r-3}(1-p)^2}{1.2.(1+\epsilon)} + \text{etc.}\right\},$$

the difference will be [436]

$$kp(1-p)\frac{r-1}{r}\int\frac{\epsilon d\epsilon}{1+\epsilon}\left[\frac{p^{r-2}}{1+\frac{r-1}{r}\epsilon} + \frac{\overline{r-2}.p^{r-3}(1-p)}{1+\frac{r-2}{r}\epsilon} + \text{etc.}\right];$$

this difference being positive, we see that there is morally the advantage to partition the sum ϵ on several vessels. This advantage is increased in measure as we increase the number r of vessels, and, if this number is very great, the moral advantage becomes nearly equal to the mathematical advantage.

In order to see this, let us take formula (a), and let us give to it this form,

$$kp\iint dx\,d\epsilon\,c^{-\left(1+\frac{\epsilon}{r}\right)x}\left(pc^{-\frac{\epsilon x}{r}} + 1 - p\right)^{r-1} + \log h; \qquad (a')$$

the integral relative to x being taken from x null to x infinity. In this interval, the coefficient of dx under the $\int\int$ signs, has neither *maximum* nor *minimum*; because its differential taken with respect to x, is

$$-c^{\left(1+\frac{\epsilon}{r}\right)x}dx\left(pc^{-\frac{\epsilon x}{r}}+1-p\right)^{r-2}\left[p(1+\epsilon)c^{-\frac{\epsilon x}{r}}+(1-p)\left(1+\frac{e}{r}\right)\right];$$

this differential is constantly negative from $x=0$ to x infinity; thus the coefficient itself diminishes constantly in this interval. It is therefore here the case to make use of formula (A) of §22 of the first Book, in order to have, by a convergent approximation, the integral $\int y\,dx,\ y$ being equal to

$$c^{-\left(1+\frac{\epsilon}{r}\right)x}\left(pc^{-\frac{\epsilon x}{r}}+1-p\right)^{r-1}.$$

The quantity that we have named ν in the section cited, becomes then

$$\nu=-\frac{y\,dx}{dy}=\frac{pc^{-\frac{\epsilon x}{r}}+1-p}{p(1+\epsilon)c^{-\frac{\epsilon x}{r}}+(1-p)\left(1+\frac{e}{r}\right)};$$

that which gives [437]

$$U=\frac{1}{1+p\epsilon+(1-p)\frac{e}{r}},$$

$$\frac{dU}{dx}=\frac{p(1-p)\epsilon^2\left(1-\frac{1}{r}\right)}{r\left[1+p\epsilon+(1-p)\frac{e}{r}\right]^2},$$

etc.;

U, $\frac{dU}{dx}$, etc. being that which ν, $\frac{d\nu}{dx}$, etc. become, when x is null. This premised, formula (A) cited, will give

$$\int dx\,c^{-\left(1+\frac{\epsilon}{r}\right)x}\left(pc^{-\frac{\epsilon x}{r}}+1-p\right)^{r-1}$$
$$=\frac{1}{1+p\epsilon+(1-p)\frac{e}{r}}\left\{1+\frac{p(1-p)\epsilon^2\left(1-\frac{1}{r}\right)}{r\left[1+p\epsilon+(1-p)\frac{e}{r}\right]}+\text{etc.}\right\}.$$

Formula (a') becomes thus, very nearly, when r is a great number,

$$k\int\frac{p\,d\epsilon}{1+p\epsilon}+\log h,$$

or

$$k\log(1+p\epsilon)+\log h.$$

Now let X be the physical fortune corresponding to this moral fortune; we have by that which precedes,

$$k\log X+\log h,$$

for the moral fortune corresponding to X; by comparing therefore these two expressions, we will have

$$X = 1 + p\epsilon.$$

In this case, the moral advantage is $p\epsilon$; it is therefore equal to the mathematical advantage.

Often the moral advantage of individuals is increased by the mean of the funds of assurance, at the same time as these funds produce to the assurers a certain benefit. Let us suppose, for example, that a merchant has a part ϵ of his fortune on a vessel of which the [438]
probability of the arrival is p; and that he assures this part, by giving a sum to the assurance company. For perfect equality between the mathematical lots of the company and of the merchant, the latter must give $(1-p)\epsilon$ for price of assurance. By representing by unity, the fortune of the merchant, independently of his expectation ϵ, his moral fortune will be by that which precedes,

$$kp \log(1+\epsilon) + \log h,$$

in the case where one does not assure; and in the case where he assures, it will be

$$k \log(1 + p\epsilon) + \log h;$$

now we have

$$\log(1 + p\epsilon) > p \log(1 + \epsilon),$$

or, that which reverts to the same,

$$\int \frac{p\, d\epsilon}{1 + p\epsilon} > \int \frac{p\, d\epsilon}{1 + \epsilon},$$

p being less than unity; the moral fortune of the merchant is therefore increased, by means of his assurance. He is able thus to make to the assurance company, a proper sacrifice to defray the expense of the establishment and to the benefit that it must make. If we name α this sacrifice, that is, if we suppose that the merchant gives to the company, for the price of his assurance, the sum $(1-p)\epsilon + \alpha$, we will have in the case of equality of the moral fortunes, when the merchant assures, and when he does not assure at all,

$$\log(1 - \alpha + p\epsilon) = p \log(1 + \epsilon);$$

that which gives

$$\alpha = 1 + p\epsilon - (1 + \epsilon)^p.$$

This is all that which the merchant can give to the company, without moral disadvantage; he will have therefore a moral advantage, by making a sacrifice less than this value of α, and at the same time, the company will have a benefit which, as we have seen, becomes certain, when its relations are very numerous. We see thence, how some establishments of this kind, well designed and sagely administered, can be assured a real benefit, by procuring [439]
advantages to the persons who negotiate with them: this is in general the end of all the exchanges; but here, by a particular combination, the exchange holds between two objects of like nature, of which one is only probable, while the other is certain.

§42. The principle of which we just made use in order to calculate the moral expectation, has been proposed by Daniel Bernoulli, in order to explicate the difference between the result of the calculus of probabilities and the indication of common sense, in the following problem. Two players A and B play at *heads* and *tails*, with the condition that A pays to B two francs, if *heads* arrives at the first trial; four francs, if it arrives at the second trial; eight francs if it arrives at the third trial, and so forth to the n^{th} trial. We demand that which B must give to A in commencing the game.

It is clear that the advantage of B, relative to the first trial, is one franc; because he has $\frac{1}{2}$ of probability to win two francs at this trial. His advantage relative to the second trial, is similarly one franc; because he has $\frac{1}{4}$ of probability to win four francs at this trial, and so forth; so that the sum of all his advantages relative to the n trials, is n francs. He must therefore for the mathematical equality of the game, give to A, this sum which becomes infinite, if we suppose that the game continues to infinity.

However a person, in this game, will not risk with prudence, an even rather moderate sum, such as one hundred francs. If we reflect in the least on this kind of contradiction between the calculus, and that which common sense indicates; we see easily that it depends on this that if we suppose, for example, $n = 50$, that which gives 2^{50} for the sum that B can hope at the fiftieth trial, this immense sum produces to B not at all, a moral advantage proportional to its magnitude; in a manner that there is for him a moral disadvantage to expose a franc in order to obtain it with the excessively small probability $\frac{1}{2^{50}}$ to succeed. But the moral advantage that an expected sum can procure, depends on an infinity of circumstances proper to each individual, and that it is impossible to evaluate. The only general consideration that we are able to employ in this regard, is that the more one is rich, [440]
the less the very small sum can be advantageous, all things equal besides. Thus the most natural supposition that we can make, is that of a reciprocal moral advantage, to the wealth of the interested person. This is to that which the principle of Daniel Bernoulli is reduced, a principle which, as we have just seen, makes the results of the calculus coincide with the indications of common sense, and which gives the means to estimate with some exactitude, these always vague indications. His application to the problem of which we have just spoke, will furnish us a new example of it.

Let us name a the fortune of B before the game, and x that which he gives to player A. His fortune becomes $a - x + 2$, if *heads* arrives at the first trial; it becomes $a - x + 2^2$, if *heads* arrives at the second trial, and so forth to trial n, where it becomes $a - x + 2^n$, if *heads* arrives only at the n^{th} trial. The fortune of B becomes $a - x$, if *heads* arrives not at all in the n trials, after which the game is supposed to end; but the probability of this last event is $\frac{1}{2^n}$. By multiplying the logarithms of these diverse fortunes by their respective probabilities and by k, we will have by that which precedes, the moral fortune of B, by virtue of the conditions of the game, equal to

$$\frac{1}{2}k\log(a-x+2) + \frac{1}{2^2}k\log(a-x+2^2) + \cdots$$
$$\cdots + \frac{1}{2^n}k\log(a-x+2^n) + \frac{1}{2^n}k\log(a-x) + \log h.$$

But before the game, his moral fortune was $k\log a + \log h$; by equating therefore these two fortunes, provided that B always conserves the same moral fortune, and passing again from the logarithms to the numbers, we will have, $a - x$ being supposed equal to a', and making $\frac{1}{a'} = \alpha$,

$$1 + \alpha x = (1 + 2\alpha)^{\frac{1}{2}}(1 + 2^2\alpha)^{\frac{1}{2^2}} \cdots (1 + 2^n\alpha)^{\frac{1}{2^n}}; \qquad (o)$$

the factors $(1 + 2\alpha)^{\frac{1}{2}}$, $(1 + 2^2\alpha)^{\frac{1}{2^2}}$ diminishing without ceasing, and their limit is unity; [441]
because we have

$$(1 + 2^i\alpha)^{\frac{1}{2^i}} > (1 + 2^{i+1}\alpha)^{\frac{1}{2^{i+1}}}.$$

In fact, if we raise to the power 2^{i+1}, the two members of this inequality, it becomes

$$1 + 2^{i+1}\alpha + 2^{2i}\alpha^2 > 1 + 2^{i+1}\alpha;$$

and under this form, the equality becomes evident. Moreover, the logarithm of $(1 + 2^i\alpha)^{\frac{1}{2^i}}$ is equal to $\frac{i\log 2}{2^i} + \frac{1}{2^i}\log\left(\alpha + \frac{1}{2^i}\right)$; and it is clear that this function is null in the case of i infinite, that which requires that in this case, $(1 + 2^i\alpha)^{\frac{1}{2^i}}$ is unity.

If we suppose n infinite in equation (o), we have the case where the game can be prolonged to infinity, that which is the most advantageous case to B. a' and consequently α being supposed known; we will take the sum of the tabular logarithms of a rather great number $i - 1$, of the first factors of the second member, in order that $2^i\alpha$ is at least equal to ten. The sum of the tabular logarithms of the following factors, to infinity, will be, very nearly equal to

$$\frac{\log\alpha}{2^{i-1}} + \frac{(i+1)\log 2}{2^{i-1}} + \frac{0,4342945}{3\alpha 2^{i-2}}.$$

The addition of these two sums will give the tabular logarithm of $a' + x$ or of a. Thus we will have for a physical fortune a, supposed B has before the game, the value of x which he must give to A at the beginning of the game, in order to conserve the same moral fortune. By supposing, for example, a' equal to one hundred, we find $a = 107^{\text{fr}}, 89$, whence it follows that the physical fortune of B being originally $107^{\text{fr}}, 89$, he must then risk prudently in this game, only $7^{\text{fr}}, 89$, instead of the infinite sum that the result of the calculus indicates, when we set aside all moral considerations. Having thus the value of a relative to $a' = 100$, it is easy to conclude from it in the following manner, its value relative to $a' = 200$; in fact we have, in this last case, [442]

$$a = (100 + 2)^{\frac{1}{2}}(100 + 2^2)^{\frac{1}{4}}.\text{etc.} = 2(100 + 1)^{\frac{1}{2}}(100 + 2)^{\frac{1}{4}}(100 + 4)^{\frac{1}{8}}.\text{etc.}$$

But we have just found

$$(100 + 2)^{\frac{1}{4}}(100 + 4)^{\frac{1}{8}}.\text{etc.} = (107, 89)^{\frac{1}{2}};$$

therefore

$$a = 2\sqrt{101.107, 89} = 208, 78.$$

Thus the physical fortune of B being originally $208, 78$, he is not able to risk prudently in this game, beyond $8^{\text{fr}}, 78$.

§43. We will now extend the principle exposed above, to the things of which the existence is distant and uncertain. For this, let us consider two persons A and B, who wish to each invest, in a life annuity, a capital q. They can make it separately: they can partner and constitute a life annuity on their heads, in a manner that the pension is reversible to the one who survives the other. Let us examine what is the most advantageous part.

Let us suppose the two persons of the same age, and having the same annual fortune that we will represent by unity, independently of the capital that they wish to place. Let β be the life pension that this capital would produce to each of them, if they placed their capitals separately, so that their annual fortune becomes $1 + \beta$. We will express, conformably to the principle of which there is concern, their corresponding annual moral fortune, by $k\log(1 + \beta) + \log h$. But this fortune will take place only probably, in the x^{th} year; thus, by designating by y_x the probability that A will survive to the end of the x^{th} year, we must multiply his annual moral fortune relative to this year, by y_x; by adding therefore all these products, their sum, that we will designate by $[k\log(1+\beta)+\log h]\sum y_x$, will be that which I name here *life-annuity moral fortune*.

Let us suppose now that A and B place the sum $2q$ of their capitals, on their heads, and that that produces a life pension β', reversible to the survivor. So long as A and B will live, each of them will touch only $\frac{1}{2}\beta'$ of life annuity, and their annual moral fortune will be $k\log(1+\frac{1}{2}\beta')+\log h$. By multiplying it by the probability that they both will live to the [443]
end of year x, a probability equal to $(y_x)^2$; the sum of these products for all the values of x, will be the life-annuity moral fortune of A, relative to the supposition of their simultaneous existence; this fortune is therefore

$$\left[k\log\left(1+\tfrac{\beta'}{2}\right)+\log h\right]\sum(y_x)^2.$$

The probability that A will exist alone to the end of the x^{th} year, is $y_x-(y_x)^2$; his life-annuity moral fortune relative to his existence after the death of B, which renders his annual moral fortune equal to $1 + \beta'$, is therefore

$$[k\log(1+\beta')+\log h]\ \sum\left[y_x-(y_x)^2\right].$$

The sum of these two functions,

$$k\log\left(1+\tfrac{\beta'}{2}\right)\sum(y_x)^2+k\log(1+\beta')\left[\sum y_x-\sum(y_x)^2\right]+\log h\sum y_x,$$

will be the life-annuity moral fortune of A under the hypothesis where A and B place conjointly their capital.

If we compare this fortune to that which we have just found in the case where they place their capitals separately; we see that there will be for A advantage or disadvantage to place conjointly, according as

$$\log\left(1+\tfrac{\beta'}{2}\right)\sum(y_x)^2+\log(1+\beta')\left[\sum y_x-\sum(y_x)^2\right]$$

will be greater or lesser than $\log(1+\beta')\sum y_x$. In order to know it, it is necessary to determine the ratio of β' to β; now we have, by §40,

$$q = \beta \sum p^x y_x,$$

$\frac{1-p}{p}$ being the annual interest on the money: we have next by the same section,

$$2q = \beta' \sum p^x \left[2y_x - (y_x)^2\right];$$

we have therefore

$$\beta' = \frac{2\beta \sum p^x y_x}{\sum p^x \left[2y_x - (y_x)^2\right]}.$$

The tables of mortality will give the values of $\sum y_x$, $\sum (y_x)^2$, $\sum p^x y_x$, $\sum p^x (y_x)^2$; we [444]
will be able thus to judge which of the two placements of which there is concern, is most advantageous.

Let us suppose β and β' some very small fractions; the quantity $\log(1+\beta)\sum y_x$ becomes very nearly $\beta \sum y_x$. The quantity

$$\log\left(1+\frac{\beta'}{2}\right)\sum (y_x)^2 + \log(1+\beta')\left[\sum y_x - \sum (y_x)^2\right]$$

becomes

$$\frac{\beta'}{2}\left[2\sum y_x - \sum (y_x)^2\right],$$

and by substituting for β' its preceding value, it becomes

$$\beta \frac{\left[2\sum y_x - \sum (y_x)^2\right]\sum p^x y_x}{2\sum p^x y_x - \sum p^x (y_x)^2};$$

there is therefore advantage to place conjointly, if

$$\left[2\sum y_x - \sum (y_x)^2\right]\sum p^x y_x$$

surpasses over

$$\left[2\sum p^x y_x - \sum p^x (y_x)^2\right]\sum y_x,$$

or if we have

$$\frac{\sum p^x (y_x)^2}{\sum p^x y_x} > \frac{\sum (y_x)^2}{\sum y_x};$$

it is in fact that which holds generally, p being smaller than unity.

The advantage to place conjointly the capitals, increases by the consideration that the increase $\frac{\beta'}{2}$ of revenue arrives to the survivor, at an ordinarily advanced age in which the greatest needs which are sensed, render it much more useful. This advantage increases yet on all the affections which the two individuals can attach to one another, and which

make them desire the well being of the one who must survive. The establishments in which one is able thus to place his capitals, and by a slight sacrifice of his revenue, to assure the existence of his family for a time where one must fear no longer being sufficient to its needs, are therefore very advantageous to the dead, by favoring the softest penchants of nature. They offer not at all the inconvenience that we have noted in even the most equitable games, the one to render the loss more sensible than the gain; since to the contrary, they [445]
offer the means to exchange the superfluous, against some assured resources in the future. The Government must therefore encourage these establishments, and to respect them in their vicissitudes; because the expectations that they present, carrying onto an extended future, they are able to prosper only with shelter from all anxiety on their duration.