NOUVELLE MÉTHODE

POUR

DÉTERMINER L'ORBITE DES COMÈTES

D'APRES LES OBSERVATIONS*

Lagrange†

Connaissance des Temps... pour l'an 1821. (1818)
Œuvres de Lagrange, Tome 7 (1877), pp. 469–483

1. The methods that one has proposed until now in order to determine the orbit of the comets, according to the observations, demand only three geocentric places observed with the intervals of time between the three observations, but suppose at the same time that the orbit of the comet is a parabola. Now, on the one side, it is very rare that one has only three observations of a comet, and, on the other, the example of the comet of 1770 proves rather that one could not adopt generally the hypothesis of the parabolic orbit. These considerations, joined to the difficulties of the methods which employ only three observations, have engaged me to examine if, by making usage of a great number of observations, one could not facilitate and generalize the solution of the Problem of the comets, and I have found the following method, which, by means of six observations, reduce the seeking of the elements of the orbit, whatever it be, to a simple equation of the seventh degree.

2. Let, in any one observation, g be the geocentric longitude of the comet, h its geocentric latitude supposed northern (one will take the angle h negative when the latitude will be southern), s the longitude of the Sun, and r the distance from the Sun to the Earth: these four quantities are known and must be taken for the data of the Problem.

Let one name now l, m, n the three rectangular coordinates which determine the apparent or geocentric place of the comet, l being the abscissa taken from the center of the Earth and parallel to the line of spring equinox, m the ordinate perpendicular to l, in the plane of the ecliptic, and n the second ordinate perpendicular to the same plane of the ecliptic; one will have, by Trigonometry, by designating by δ the unknown distance of the comet to the Earth,

$$l = \delta \cos h \cos g, \qquad m = \delta \cos h \sin g, \qquad n = \delta \sin h,$$

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and, if one names moreover p and q the abscissa and the ordinate of the place of the Sun, one will have likewise

$$p = r\cos s, \qquad q = r\sin s.$$

Finally, if one names x, y, z the rectangular coordinates of the heliocentric place of the comet, that is x the abscissa taken from the center of the Sun, and parallel to the line of the spring equinox, y the ordinate perpendicular to x in the plane of the ecliptic, and z the ordinate perpendicular to the same plane of the ecliptic, so that the lines x, y, z are respectively parallel to the lines l, m, n, it is easy to understand that one will have

$$x = l - p, \qquad r = m = q, \qquad z = n;$$

therefore

$$\begin{aligned} x &= \delta\cos h\cos g - r\cos s,\\ y &= \delta\cos h\sin g - r\sin s,\\ z &= \delta\sin h. \end{aligned}$$

3. Let now γ be the longitude of the ascendant node of the orbit of the comet, and η the inclination of the plane of this orbit on the plane of the ecliptic, this angle η being counted formed in the eastern and northern part of the sphere. The general equation of the plane of the orbit of the comet will be of this form

$$z = \tan\eta\ \cos\gamma.y - \tan\eta\ \sin\gamma.x,$$

or else, by making, for more simplicity, $\tan\eta\ \sin\gamma = \alpha$, $\tan\eta\ \cos\gamma = \beta$,

$$z = \beta y - \alpha x.$$

It is this which is known rather by the theory of curves.

4. Let one substitute therefore in the preceding equation, for x, y, z, their values found above (2), one will have this here

$$\delta\sin h = \beta\delta\cos h\ \sin g - \beta r\sin s - \alpha\delta\cos h\ \cos g + \alpha r\cos s,$$

whence one draws

$$\delta = r\frac{\alpha\cos s - \beta\sin s}{\sin h + \alpha\cos h\ \cos g - \beta\cos h\ \sin g};$$

setting this value into the same expressions of x, y, z, one will have

$$\begin{aligned} x &= r\frac{\beta\sin(g-s) - \tan h\ \cos s}{\tan h + \alpha\cos g - \beta\sin g},\\ y &= r\frac{\alpha\sin(g-s) - \tan h\ \sin s}{\tan h + \alpha\cos g - \beta\sin g},\\ z &= r\frac{\alpha\tan h\ \cos s - \beta\tan h\ \sin s}{\tan h + \alpha\cos g - \beta\sin g}, \end{aligned}$$

In these expressions there is, as one sees, unknown only the two quantities α and β, which depend on the position of the plane of the orbit of the comet on the ecliptic.

5. We suppose that, in another observation, the quantities g, h, r, s, x, y, z become g', h', r', s', x', y', z'; one will have likewise

$$\begin{aligned} x' &= r'\frac{\beta\sin(g'-s') - \tan h'\ \cos s'}{\tan h' + \alpha\cos g' - \beta\sin g'}, \\ y' &= r'\frac{\alpha\sin(g'-s') - \tan h'\ \sin s'}{\tan h' + \alpha\cos g' - \beta\sin g'}, \\ z' &= r'\frac{\alpha\tan h'\ \cos s' - \beta\tan h'\ \sin s'}{\tan h' + \alpha\cos g' - \beta\sin g'}, \end{aligned}$$

and, if one substitutes the preceding values of x, y, x', y' in the expression $y'x - x'y$, one will find the quantity

$$rr'\frac{\mathrm{G}\alpha - \mathrm{H}\beta + \tan h' \tan gh \sin(s'-s)}{(\tan h' + \alpha\cos g' - \beta\sin g')(\tan h + \alpha\cos g - \beta\sin g)},$$

by making, for brevity,

$$\begin{aligned} \mathrm{G} &= \tan h'\ \cos s'\ \sin(g-s) - \tan h\ \cos s\ \sin(g'-s'), \\ \mathrm{H} &= \tan h'\ \sin s'\ \sin(g-s) - \tan h\ \sin s\ \sin(g'-s'), \end{aligned}$$

6. Now it is easy to be convinced, by Geometry, that $\frac{y'x-x'y}{2}$ is equal to the area of the triangle which is the projection on the ecliptic of the triangle formed in the plane of the orbit of the comet by the two radius vectors drawn from the Sun to the two places observed, and the straight line chord which joins these two places and which subtends, consequently, the arc traversed in the interval of the two observations; moreover, if one names Δ the area of this last triangle, one will prove easily, by Geometry, that $\Delta : \frac{y'x-x'y}{2} = 1 : \cos\eta$, η being the inclination of the plane of the orbit on the one of the ecliptic (3), so that one will have

$$\Delta = \frac{y'x - x'y}{2\cos\eta};$$

but

$$\tan\eta = \sqrt{\alpha^2+\beta^2},$$

and consequently

$$\cos\eta = \frac{1}{\sqrt{1+\alpha^2+\beta^2}};$$

therefore one will have, after the substitutions,

$$\Delta = \frac{[\mathrm{G}\alpha - \mathrm{H}\beta + \tan h'\ \tan h \sin(s'-s)]rr'\sqrt{1+\alpha^2+\beta^2}}{2(\tan h' + \alpha\cos g' - \beta\sin g')(\tan h + \alpha\cos g - \beta\sin g)}.$$

7. Now, as s is the longitude of the Sun in the time of the first observation, and s' its longitude in the time of the second observation, it follows that $s' - s$ will be the angle described by the Sun about the Earth, or, that which reverts to the same, the angle described by the Earth about the Sun, in the interval of the two observations; but r is the distance from the Earth to the Sun in the first observation, and r' the distance from the Earth to the Sun in the second observation; therefore the area of the triangle, formed at the center of the Sun by the two radius vectors r, r', and by the chord of the arc traversed by the Earth in the interval of the two observations, will be expressed by $\frac{rr'\sin(s'-s)}{2}$, as one demonstrates it in Geometry.

Therefore the ratio of the triangular area Δ, described by the comet in the interval of two observations, to the triangular area described by the Earth in the same time, will be expressed by $\frac{2\Delta}{rr'\sin(s'-s)}$, that is to say, by substituting the value above of Δ, by making

$$\begin{aligned}
\mathrm{A} &= \frac{\tan h' \ \cos s' \ \sin(g-s) - \tan h \ \cos s \sin(g'-s')}{\sin(s'-s)}, \\
\mathrm{B} &= \frac{\tan h' \ \sin s' \ \sin(g-s) - \tan h \ \sin s \sin(g'-s')}{\sin(s'-s)},
\end{aligned}$$

by the formula

$$\frac{\tan h \ \tan h' + \mathrm{A}\alpha - \mathrm{B}\beta)\sqrt{1+\alpha^2+\beta^2}}{(\tan h + \alpha\cos g - \beta\sin g)(\tan h' + \alpha\cos g' - \beta\sin g')}.$$

8. I remark now that, if the interval between the two observations is quite small, the arcs traversed by the comet and by the Earth about the Sun will be confounded very nearly with the chords, and consequently the triangular sectors, of which we just determined the ratio, will be able to be taken, without sensible error, for the true curvilinear sectors described by the comet and by the Earth. Now one knows, by the theory of central forces, that, in the conic sections described about one same focus and by one same force, which varies in inverse ratio of the square of the distances, the areas of the sectors described in the same time are between them as the square roots of the parameters of the sections; therefore, if one names ϖ the half-parameter of the ellipse described by the comet about the Sun, and Π the half-parameter of the orbit of the Earth, which is very nearly equal to the mean distance from the Sun, one will have, under the hypothesis that the two observations are very close, the equation

$$\frac{\tan h \ \tan h' + \mathrm{A}\alpha - \mathrm{B}\beta}{(\tan h + \alpha\cos g - \beta\sin g)(\tan h' + \alpha\cos g' - \beta\sin g')} = \sqrt{\frac{\varpi}{(1+\alpha^2+\beta^2)\Pi}},$$

which will be so much more exact as the interval between the two observations will be smaller.

9. The preceding equation contains, as one sees, three unknowns α, β and ϖ; thus it will be necessary three similar equations in order to determine these unknowns, and, as the second member remains the same, one will eliminate first the unknown ϖ by simply subtracting one equation from the other, and one will have two equations

between the two unknowns α and β, in which these unknowns will rise only to the third degree; so that the final equation in α and β will not be able to surpass the ninth degree. Having found the values of α and β, one will know first the position of the plane of the orbit of the comet (3); next one of the three equations will give the value of the parameter ϖ, and one will determine the other elements by the known methods.

This method demands therefore six observations of the comet, made in a manner that the intervals of time, between the first and the second, between the third and the fourth, between the fifth and the sixth, are very small; but, for the intervals between the second and the third, and between the fourth and the fifth, they are able to be any, and it will be even advantageous to take them the greatest that one is able, so that the three equations are the most different that it is possible.

10. By taking, as we suppose it, the triangular sectors in place of the true curvilinear sectors described by the comet and by the Earth, in the interval of the two observations, one neglects the segments formed by the arcs traversed by the comet and by the Earth and by the the chords which subtend these arcs; now, when the arcs are very small of the first order, the sectors are also very small of the first order; but the segments become very small of the third order, because the chords are very small of the first order, and because the versines are very small of the second order. Therefore, under this hypothesis, the ratio of the triangular sectors of the comet and of the Earth differ from the ratio of the curvilinear sectors only by some quantities of the second order only; consequently, in the equation found above (8), the first member will be exact, nearly with the very small quantities of the second order, by regarding the differences of the quantities which are related to the two observations as very small of the first order; thus one will be able, without altering the exactness of the equation of which there is concern, to neglect, in its first member, the quantities in which the very small arcs $s'-s$, $g'-g$, $h'-h$ would form products of two or a great number of dimensions.

This remark is able to serve to render the equation of which we just spoke a little more simple. In fact, it is clear that one is able to set the quantity $\tan h + \alpha\cos g - \beta\sin g$ under this form

$$\begin{aligned} &\frac{\tan h' + \tan h}{2} + \alpha\cos\frac{g'+g}{2}\cos\frac{g'-g}{2} - \beta\sin\frac{g'+g}{2}\cos\frac{g'+g}{2} \\ - \quad &\frac{\tan h' - \tan h}{2} + \alpha\sin\frac{g'+g}{2}\sin\frac{g'-g}{2} - \beta\cos\frac{g'+g}{2}\sin\frac{g'+g}{2}, \end{aligned}$$

and likewise the quantity $\tan h' + \alpha\cos g' - \beta\sin g'$ under the form

$$\begin{aligned} &\frac{\tan h' + \tan h}{2} + \alpha\cos\frac{g'+g}{2}\cos\frac{g'-g}{2} - \beta\sin\frac{g'+g}{2}\cos\frac{g'-g}{2} \\ + \quad &\frac{\tan h' - \tan h}{2} - \alpha\sin\frac{g'+g}{2}\sin\frac{g'-g}{2} - \beta\cos\frac{g'+g}{2}\sin\frac{g'-g}{2}; \end{aligned}$$

so that the product of these two quantities will become

$$\begin{aligned} &\left[\frac{\tan h' + \tan h}{2} + \left(\alpha\cos\frac{g'+g}{2} - \beta\sin\frac{g'+g}{2}\right)\cos\frac{g'-g}{2}\right]^2 \\ + \quad &\left[\frac{\tan h' - \tan h}{2} - \left(\alpha\sin\frac{g'+g}{2} + \beta\cos\frac{g'+g}{2}\right)\sin\frac{g'-g}{2}\right]^2, \end{aligned}$$

an expression which, by neglecting the very small quantities of the second order, is reduced to this here

$$\left(\frac{\tan h' \ \tan h}{2} + \alpha \cos\frac{g'+g}{2} - \beta \sin\frac{g'+g}{2}\right)^2,$$

that one will be able to substitute in the place of the denominator of the first member of the equation of n° 8.

One would be able to make parallel reductions in the numerator of the first member of the same equation; but I am myself assured by the calculations that the values of A and B would not become simpler.

11. Retaining therefore the expressions of the coefficients A and B, given in n° 7, and making moreover

$$\mathrm{C} = \tan h \ \tan h', \qquad a = \cos\frac{g'+g}{2}, \qquad b = \sin\frac{g'+g}{2}, \qquad c = \frac{\tan h' + \tan h}{2},$$

the equation of n° 8 will become

$$\frac{\mathrm{A}\alpha - \mathrm{B}\beta + \mathrm{C}}{(a\alpha - b\beta + c)^2} = \sqrt{\frac{\varpi}{(1+\alpha^2+\beta^2)\Pi}},$$

and each pair of observations of the comet, of which the interval is very small, will furnish a parallel equation, in which the coefficients A, B, C, a, b, c will be known; therefore, if one has three pairs of such observations, one will deduce from them three equations of the form

$$\begin{aligned}
\frac{\mathrm{A}_1\alpha - \mathrm{B}_1\beta + \mathrm{C}_1}{(a_1\alpha - b_1\beta + c_1)^2} &= \sqrt{\frac{\varpi}{(1+\alpha^2+\beta^2)\Pi}},\\
\frac{\mathrm{A}_2\alpha - \mathrm{B}_2\beta + \mathrm{C}_2}{(a_2\alpha - b_2\beta + c_2)^2} &= \sqrt{\frac{\varpi}{(1+\alpha^2+\beta^2)\Pi}},\\
\frac{\mathrm{A}_3\alpha - \mathrm{B}_3\beta + \mathrm{C}_3}{(a_3\alpha - b_3\beta + c_3)^2} &= \sqrt{\frac{\varpi}{(1+\alpha^2+\beta^2)\Pi}},
\end{aligned}$$

which will serve to determine the three unknowns α, β and ϖ.

These equations give first these two here

$$\begin{aligned}
\frac{\mathrm{A}_1\alpha - \mathrm{B}_1\beta + \mathrm{C}_1}{(a_1\alpha - b_1\beta + c_1)^2} &= \frac{\mathrm{A}_2\alpha - \mathrm{B}_2\beta + \mathrm{C}_2}{(a_2\alpha - b_2\beta + c_2)^2},\\
\frac{\mathrm{A}_1\alpha - \mathrm{B}_1\beta + \mathrm{C}_1}{(a_1\alpha - b_1\beta + c_1)^2} &= \frac{\mathrm{A}_3\alpha - \mathrm{B}_3\beta + \mathrm{C}_3}{(a_3\alpha - b_3\beta + c_3)^2},
\end{aligned}$$

by which one will be able to determine the two unknowns α and β.

Let us suppose

$$\begin{aligned}
\mathrm{A}_1\alpha - \mathrm{B}_1\beta + \mathrm{C}_1 &= x,\\
a_1\alpha - b_1\beta + c_1 &= y,
\end{aligned}$$

one will have

$$\alpha = \frac{(x-c_1)b_1 - (y-c_1)\mathrm{B}_1}{\mathrm{A}_1 b_1 - a_1\mathrm{B}_1},$$
$$\beta = \frac{(x-c_1)a_1 - (y-c_1)\mathrm{A}_1}{\mathrm{A}_1 b_1 - a_1\mathrm{B}_1};$$

the first member of the two preceding equations will become $\frac{x}{y^2}$; and, substituting the preceding values of α and β into the second members of the same equations, they will become evidently of the form

$$\frac{x}{y^2} = \frac{lx+my+n}{(Lx+My+N)^2},$$
$$\frac{x}{y^2} = \frac{px+qy+r}{(Px+Qy+R)^2};$$

now these equations are changed into these here

$$\frac{x}{y} = \frac{l\frac{x}{y}+m+\frac{n}{y}}{\left(L\frac{x}{y}+M+\frac{N}{y}\right)^2},$$
$$\frac{x}{y} = \frac{p\frac{x}{y}+q+\frac{r}{y}}{\left(P\frac{x}{y}+Q+\frac{R}{y}\right)^2};$$

namely, by making $\frac{x}{y} = t$, $\frac{1}{y} = u$,

$$t = \frac{lt+m+nu}{(Lt+M+Nu)^2},$$
$$t = \frac{pt+q+ru}{(Pt+Q+Ru)^2}.$$

Let us suppose moreover

$$Lt+M+Nu = z,$$

one will have

$$u = \frac{z-Lt-M}{N},$$

and, making this substitution into the preceding equations, they will take this form

$$t = \frac{l't+m'+n'z}{z^2},$$
$$t = \frac{p't+q'+r'z}{(P't+Q'+R'z)^2};$$

the first gives immediately

$$t = \frac{m'+n'z}{z^2-l'},$$

and, this value being substituted into the second, one will have this final equation in z,

$$\frac{m' + n'z}{z^2 - l'} = \frac{p'(m' + n'z)(z^2 - l') + (q' + r'z)(z^2 - l)^2}{[P'(M' + N'z) + (Q' + R'z)(z^2 - l')]^2},$$

which, being developed and ordered with respect to z, will rise to the seventh degree, and will have consequently always at least one real root.

12. It will not be difficult to resolve by approximation the equation that we just found, and for that it will be yet better to use the two equations

$$t = \frac{m' + n'z}{z^2 - l'} \qquad \text{and} \qquad (\mathrm{P}'t + \mathrm{Q}' + \mathrm{R}'z)^2 = \frac{p't + q' + r'z}{t};$$

one will give successively to z different values, and one will calculate those of t and of the two quantities $\mathrm{P}'t + \mathrm{Q}' + \mathrm{R}'z$, $\frac{p't+q'+r'z}{t}$; if one finds two values of z, of which one renders the second of these quantities greater than the square of the first, and of which the other renders it smaller, one will be assured that the true value of z falls between these two, and one will be able next, by other substitutions, to approach more this value. If one was not able to encounter two substitutions which give results of contrary signs, he would have then only to operate according to the rules that I have given in my Memoir on the resolution of numeric equations (*Mémoires de* 1767), and by which one is always certain to discover and to determine as exactly as one wishes all the real roots of any equation.

Having found a convenient value of z, one will have those of t and of u, next those of $x = \frac{t}{u}$ and of $y = \frac{1}{u}$, next those of α and β, which will make known first the position of the orbit (3), and one will know also at the same time the value of the parameter ϖ by means of the equation

$$\sqrt{\frac{\varpi}{(1 + \alpha^2 + \beta^2)\Pi}} = \frac{x}{y^2} = tu.$$

13. Knowing α and β, one will know, if one wishes, the distance δ from the comet to the Earth, thus as the coordinates x, y, z of the position of the comet in its orbit, for each of the six observations, by means of the formulas of n° 4; one will know therefore also the radius vector of the comet, that I will designate by v, and which is $= \sqrt{x^2 + y^2 + z^2}$.

Now the equation of any conic section reported to the focus is

$$\varpi - v = \mu x + \nu y,$$

in which ϖ is the half-parameter, and μ, ν are two constants such that, if one names ϵ the excentricity and ω the anomaly which corresponds to the ascendant node, that is to say to the distance of the node to the perihelion of the orbit, one has

$$\begin{aligned} \mu &= \epsilon\left(\cos\gamma\ \cos\omega + \frac{\sin\gamma\ \sin\omega}{\cos\eta}\right), \\ \nu &= \epsilon\left(\sin\gamma\ \cos\omega + \frac{\cos\gamma\ \sin\omega}{\cos\eta}\right), \end{aligned}$$

I give not at all the demonstration of these formulas, because it would draw me too far, and because it is besides not difficult to find it, according to the known properties of the conic sections.

Substituting therefore into the equation $\varpi - v = \mu x + \nu y$ the values of x, y and v, which correspond to any two observations, as to the first and to the last, in order to take the most distant that it is possible, one will have two equations, by means of which one will determine immediately the values of μ and of ν, since that of ϖ is already known.

Having the values of μ and of ν, one will know the angle ω through the equation

$$\frac{\cos\gamma\ \cos\eta + \sin\gamma\ \tan\omega}{\sin\gamma\ \cos\eta + \cos\gamma\ \tan\omega} = \frac{\mu}{\nu},$$

and the eccentricity ϵ through the equation

$$\epsilon = \frac{\sqrt{\mu^2 + \nu^2}}{\sqrt{\sec^2\eta - \tan^2\eta\cos^2\omega}},$$

the angles η and γ being known by means of the quantities α and β (3).

If the orbit is parabolic, or very nearly, the value of ϵ will be exactly, or very nearly, equal to unity; if not one will have

$$\epsilon = \sqrt{1 - \frac{\varpi}{\lambda}},$$

λ being the mean distance or the semi-major axis of the orbit.

Finally, γ being the longitude of the ascendant node, and ω the distance from the node to the perihelion, one will have $\gamma - \omega$ for the longitude of the perihelion.

14. Now, if one names θ the mean movement from the Sun during the time elapsed between the passage of the comet through the perihelion and any of the observations, in which the vector radius of the comet is $= v$, one will have, by taking the mean distance from the Sun for unity, and making, for brevity,

$$\cos\phi = \frac{1 - \frac{v}{\lambda}}{\epsilon},$$

one will have, I say, by the known formulas,

$$\theta = (\phi - \epsilon\sin\phi)\sqrt{\lambda^3},$$

where ϕ will be at the same time that which one names, following Kepler, the eccentric anomaly.

And when the semi-major axis λ will be quite large, thus as it takes place in the orbit of comets, one will have, by the series,

$$\theta = \frac{\varpi}{1+\epsilon}\psi + \frac{1}{6}\psi^3 + \frac{3}{20\lambda}\psi^5 + \frac{5}{112\lambda}\psi^7 + \cdots,$$

by making

$$\psi = \frac{\sqrt{2v - \varpi - \frac{v^2}{\lambda}}}{\epsilon}.$$

In the parabola, where $\lambda = \infty$, and consequently $\epsilon = 1$, one will have simply

$$\psi = \sqrt{2v - \varpi}, \quad \text{and} \quad \theta = \frac{\varpi}{2}\psi + \frac{1}{6}\psi^2;$$

and $\frac{\varpi}{2}$ will be then equal to the perihelion distance.

One will know therefore, by the preceding formulas, the time of the passage of the comet through the perihelion, and, if one calculates this time according to two observations separated enough, one will be able, through the accord of the results, to judge the exactitude of the elements of the orbit found by the method exposed above; one will be able also by this means to correct these same elements, in the case that they are not exact enough; finally this calculation will serve to make known which of the roots of equation z (11) it will be necessary to employ, if it happens that it has more than one of them real.

15. The method that we just exposed in this Memoir is perhaps one of the simplest and most sure that one may imagine in order to resolve directly and without groping the famous Problem of the determination of the orbit of comets according to observations. Beyond that it requires only the resolution of one equation of the seventh degree, it has yet the advantage to be equally applicable, be it that the orbit of the comet is a parabola, or any other conic section.

In regard to the six observations that it demands, if, among those which will have been made, there is not found which had the requisite condition, that is to say which were two-by-two nearby, it would be always easy to supplement by the known method of interpolation, and likewise it will be always appropriate to employ this method in order to correct the immediate results of the observations.

I must not fail to remark, in finishing, that, when one wishes to resolve directly and rigorously the Problem of the comets by means of three nearby observations and under the parabolic hypothesis, one is also led to an equation of the seventh degree, thus as I have shown in my researches on this subject (*Mémoires de* 1778)[1] so that it seems that the seventh degree is a limit below which it is not possible to reduce the Problem of which there is concern, in any fashion that one envisions it. Besides, although the method of this Memoir demands also some quite near observations, it is however easy to be convinced that it is much more certain than that of the Researches cited, since one considers the movement of the comet in three parts of the orbit, in truth infinitely small or regarded as such, but at the same time quite different from one another, instead that, in the other method, one determines the orbit only according to two infinitely small and consecutive parts, or, that which reverts to the same, after a single infinitely small portion of the orbit.

[1] *Œuvres de Lagrange*, T. IV, p. 439.