

**ENCYCLOPÉDIE OU *DICTIONNAIRE* RAISONNÉ
DES SCIENCES, DES ARTS ET DES MÉTIERS**

UNSIGNED

PHARAON

PHARAON, (*Game of chance*.) The principle rules of this game are,

That the banker deals with a whole deck composed of fifty-two cards.

That he draws all the cards in succession, putting some to his right, & others to his left.

That with each hand one deals, that is to say, of cards two by two: the punter¹ has the liberty of taking one or several cards, & of risking a certain sum on them.

That the banker wins the stake from the punter, when the card of the punter arrives to the right hand on an odd rank, & that he loses, when the card of the punter falls to the left hand, & in an even rank.

That the banker takes the half of that which the punter has staked on the card, when in one same deal, the card of the punter occurs two times; that which makes a part of the advantage of the banker.

And finally that the last card which should be for the punter, is neither for him, nor for the banker; that which is again an advantage for the banker.

Whence one sees, 1° that the card of the punter being not more than once in the stock, the difference of the lot of the banker & of the punter is founded on that which enters all the various possible arrangements of the cards of the banker, there are a greater number of them which make him win, than there are of them which make him lose, the last card being considered as null; 2°. that the advantage of the banker increases in measure as the number of cards of the banker decreases; 3° that the card of the punter being twice in the stock, the advantage of the banker is drawn from the probability that there is that the card of the punter will come twice in one same deal; for then the banker wins half of the stake of the punter, excepting only the case where the card of the punter would come in doublet in the last deal, that which would give to the banker the entire stake of the punter; [4°] that the card of the punter being three or four times in the hand of banker, the advantage of banker is founded on the possibility that there is that the card of the punter is found twice in a same deal, before it has come as pure gain or as pure loss for the banker. Now this possibility increases or decreases, according as there are more or less cards in the hand of the banker, & according as the card of the punter is found there more or less times.

Whence one concludes further that in order to know the advantage of the banker, with respect to the punters, in all the different circumstances of the game, it is necessary to discover in all the different possible arrangements of the cards that the banker holds, & under the assumption that the card is found there either one, or two, or three, or four times, which are those which make him win, which are those which give to him half the stake of

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¹*Translator's note:* *Punter*: the one who wagers against the banker.

the punter, which are those which make him lose, & which are those finally which make him neither lose nor win.

One is able to form two tables of all these different chances. In order to know the usage of it, in the first, the number contained in the cell \square would express the number of cards that the banker holds, & the number which follow, either the cell in the first column, or two points in the other columns, would express the number of times that the card of the punter is supposed to be found in the hand of banker.

The usage of the second table would be to give some expressions, in truth less exact, but simpler & more intelligible to the players: in order to understand this table, it is necessary to know that this sign $>$ marks excess, & that this one $<$ marks defect; so that $> 1/4 < 1/3$ means greater than $1/4$, & smaller than $1/3$.

In examining these tables, one would see in the first column that the advantage to the banker is expressed in the first column by a fraction of which the numerator being always the unit, the denominator is the number of cards that the banker holds.

In the second column, that this advantage is expressed by a fraction of which the numerator being according to the sequence of natural numbers, 1, 2, 3, 4, &c. the denominator has for difference among these terms, the numbers 8, 26, 34, 42, 50, 58, which the difference is 8.

That in the third column the numerator always being 3, the difference which rules in the denominator is 8.

That in the fourth column the difference always being 4 in the numerator, the denominator has for difference among its terms the numbers 24, 40, 56, 72, 88, & of which the difference is 16.

That another rather singular uniformity between the last digits of the denominator of each term of one column, it is that in the first the last digits of the denominator are according to this order: 4, 6, 8, 0, 2, | 4, 6, 8, 0, 2; & in the second according to this order, 2, 0, 6, 0, 2, | 2, 0, 6, 0, 2, | 2, 0, 6, 0, 2; & in the third according to this order, 2, 0, 8, 6, 4, | 2, 0, 8, 6, 4; & in the fourth according to this order, 6, 0, 0, 6, 8, | 6, 0, 0, 6, 8, &c.

One may, by the means of these tables, find all at once how much advantage a banker has on each card, how much each complete deal will have due, with equal fortune, to bring profit to the banker, if one remembers the number of cards taken by the punters, of the various circumstances in which one has staked them on the game, & finally of the quantity of money risked on these cards.

One would give some fair limits to this advantage, by establishing that the doublets are indifferent for the banker & for the punter, or at least that they are worth only to the banker the third or the quarter of the stake of the punter.

So that the punter taking a card has the least possible disadvantage, it is necessary that he chooses one of them which has passed twice; there would be greater disadvantage for him, if he would take a card which has passed once; more still on a card which has been passed three times, & the worst choice would be a card which would have not yet passed.

So, by supposing $A =$ one pistole,² the advantage of the banker who would be 19 sols³ 2 deniers, under the assumption that the card of punter would be four times in twelve cards, will become 16 sols 8 deniers, if it is there only once; 13 sols 7 deniers if it is there three times; & 10 sols 7 deniers if it is there only twice.⁴

²Translator's note: The following conversion may be used: 1 pistole = 200 sols, 1 sol = 12 deniers.

³Translator's note: In the version published in the *Encyclopédi Méthodique*, the word coin *sol* is replaced by *sou*.

⁴Translator's note: Based on the conversions given in the previous note and using the ratios given in the first table in row 12, we have

The persons who have not examined the substance of the game will ask why one has said nothing of the masses, of the parolis, of the paix, & of the sept & the *va*, it is that all this signifies nothing, that one risks more or less, & then this is all; the odds do not change at all.

The advantage of banker increases in proportion as the number of his cards decreases.

The advantage of banker on a card which has not passed, is nearly double of the one that he has on a card which has passed twice; his advantage on a card which has passed three times is to his advantage on a card which has passed twice in a greater ratio than of three to two.

The advantage of the banker which would be only about 24 sols, if the punter staked six pistoles either at the first deal of game, or on a card which would have passed twice, when there would remain no more than twenty-eight in the hand of banker (for these two cases return near enough to the same thing) will be 7 livres 2 sols, if the punter wagers six pistoles on a card which has not passed yet, the stock being composed only of ten cards.

The advantage of banker would be precisely six livres, if the card of the punter, in this last case, passes three times.

So, all the knowledge of *pharaon* is reduced for the punters to the observation of the two following rules.

To take some cards only in the first deals & wager on the game so much less, as there are a greater number of deals passed.

To regard as the worst cards those which have not yet passed, or which have passed three times, & to prefer to all those which have passed twice.

It is thus that the punter will render his disadvantage the least possible.

the case of 4 cards: $\frac{19}{198} \times 200 = 19.1919 \dots$ sols; $0.1919 \dots \times 12 = 2.3$ denier.
the case of 1 card: $\frac{1}{12} \times 200 = 16.666 \dots$ sols; $0.666 \dots \times 12 = 8$ denier.
the case of 3 cards: $\frac{3}{44} \times 200 = 13.6363 \dots$ sols; $0.6363 \dots \times 12 = 7.64$ denier.
the case of 2 cards: $\frac{7}{132} \times 200 = 10.6060 \dots$ sols; $0.6060 \dots \times 12 = 7.3$ denier.

Translator's note: These are the tables referenced by the text. They have been taken from Montmort's *Essay D'analyse sur les jeux de hazard*, Second edition, 1713. The page numbers refer to this work.

TABLE FOR PHARAON

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52	$1 = * * *$	$: 2 = * * *$	$: 3 = * * *$	$: 4 = a + \frac{2295086253}{115890841950}a$
50	$1 = * * *$	$: 2 = a + \frac{3117}{350350}a$	$: 3 = a + \frac{3}{196}a$	$: 4 = a + \frac{8208829}{349595300}a$
48	$1 = a + \frac{1}{48}a$	$: 2 = a + \frac{1787}{161304}a$	$: 3 = a + \frac{3}{188}a$	$: 4 = a + \frac{276199}{12842280}a$
46	$1 = a + \frac{1}{46}a$	$: 2 = a + \frac{3431}{296010}a$	$: 3 = a + \frac{3}{180}a$	$: 4 = a + \frac{11002}{489555}a$
44	$1 = a + \frac{1}{44}a$	$: 2 = a + \frac{822}{67639}a$	$: 3 = a + \frac{3}{172}a$	$: 4 = a + \frac{913}{38786}a$
42	$1 = a + \frac{1}{42}a$	$: 2 = a + \frac{3145}{246246}a$	$: 3 = a + \frac{3}{164}a$	$: 4 = a + \frac{79}{3198}a$
40	$1 = a + \frac{1}{40}a$	$: 2 = a + \frac{1501}{111540}a$	$: 3 = a + \frac{3}{156}a$	$: 4 = a + \frac{25}{962}a$
38	$1 = a + \frac{1}{38}a$	$: 2 = a + \frac{2849}{201068}a$	$: 3 = a + \frac{3}{148}a$	$: 4 = a + \frac{1349}{49210}a$
36	$1 = a + \frac{1}{36}a$	$: 2 = a + \frac{679}{45045}a$	$: 3 = a + \frac{3}{140}a$	$: 4 = a + \frac{1139}{39270}a$
34	$1 = a + \frac{1}{34}a$	$: 2 = a + \frac{2573}{160446}a$	$: 3 = a + \frac{3}{132}a$	$: 4 = a + \frac{357}{11594}a$
32	$1 = a + \frac{1}{32}a$	$: 2 = a + \frac{1215}{70928}a$	$: 3 = a + \frac{3}{124}a$	$: 4 = a + \frac{177}{5394}a$
30	$1 = a + \frac{1}{30}a$	$: 2 = a + \frac{2287}{124410}a$	$: 3 = a + \frac{3}{116}a$	$: 4 = a + \frac{55}{1566}a$
28	$1 = a + \frac{1}{28}a$	$: 2 = a + \frac{536}{27027}a$	$: 3 = a + \frac{3}{108}a$	$: 4 = a + \frac{221}{5850}a$
26	$1 = a + \frac{1}{26}a$	$: 2 = a + \frac{2001}{92950}a$	$: 3 = a + \frac{3}{100}a$	$: 4 = a + \frac{611}{14950}a$
24	$1 = a + \frac{1}{24}a$	$: 2 = a + \frac{929}{39468}a$	$: 3 = a + \frac{3}{92}a$	$: 4 = a + \frac{473}{10626}a$
22	$1 = a + \frac{1}{22}a$	$: 2 = a + \frac{1715}{66066}a$	$: 3 = a + \frac{3}{84}a$	$: 4 = a + \frac{143}{2926}a$
20	$1 = a + \frac{1}{20}a$	$: 2 = a + \frac{1572}{54340}a$	$: 3 = a + \frac{3}{76}a$	$: 4 = a + \frac{35}{646}a$
18	$1 = a + \frac{1}{18}a$	$: 2 = a + \frac{1429}{43758}a$	$: 3 = a + \frac{3}{68}a$	$: 4 = a + \frac{31}{510}a$
16	$1 = a + \frac{1}{16}a$	$: 2 = a + \frac{429}{11440}a$	$: 3 = a + \frac{3}{60}a$	$: 4 = a + \frac{9}{130}a$
14	$1 = a + \frac{1}{14}a$	$: 2 = a + \frac{44}{1001}a$	$: 3 = a + \frac{3}{52}a$	$: 4 = a + \frac{23}{286}a$
12	$1 = a + \frac{1}{12}a$	$: 2 = a + \frac{7}{132}a$	$: 3 = a + \frac{3}{44}a$	$: 4 = a + \frac{19}{198}a$
10	$1 = a + \frac{1}{10}a$	$: 2 = a + \frac{5}{15}a$	$: 3 = a + \frac{3}{36}a$	$: 4 = a + \frac{5}{42}a$
8	$1 = a + \frac{1}{8}a$	$: 2 = a + \frac{2}{56}a$	$: 3 = a + \frac{3}{28}a$	$: 4 = a + \frac{11}{70}a$
6	$1 = a + \frac{1}{6}a$	$: 2 = a + \frac{2}{15}a$	$: 3 = a + \frac{3}{20}a$	$: 4 = a + \frac{7}{30}a$
4	$1 = a + \frac{1}{4}a$	$: 2 = a + \frac{1}{4}a$	$: 3 = a + \frac{3}{12}a$	$: 4 = a + \frac{1}{2}a$

TABLE II. FOR PHARAON.

52	$1 = * * *$	$2 = * * *$	$3 = * * *$	$4 = a + > \frac{1}{51} < \frac{1}{50}$
50	$1 = * * *$	$2 = a + > \frac{1}{95} < \frac{1}{94}$	$3 = a + > \frac{1}{66} < \frac{1}{65}$	$4 = a + > \frac{1}{49} < \frac{1}{48}$
48	$1 = a + \frac{1}{48}a$	$2 = a + > \frac{1}{91} < \frac{1}{90}$	$3 = a + > \frac{1}{63} < \frac{1}{62}$	$4 = a + > \frac{1}{47} < \frac{1}{46}$
46	$1 = a + \frac{1}{46}a$	$2 = a + > \frac{1}{87} < \frac{1}{86}$	$3 = a + \frac{1}{60}$	$4 = a + > \frac{1}{45} < \frac{1}{44}$
44	$1 = a + \frac{1}{44}a$	$2 = a + > \frac{1}{83} < \frac{1}{82}$	$3 = a + > \frac{1}{58} < \frac{1}{57}$	$4 = a + > \frac{1}{43} < \frac{1}{42}$
42	$1 = a + \frac{1}{42}a$	$2 = a + > \frac{1}{79} < \frac{1}{78}$	$3 = a + > \frac{1}{55} < \frac{1}{54}$	$4 = a + > \frac{1}{41} < \frac{1}{40}$
40	$1 = a + \frac{1}{40}a$	$2 = a + > \frac{1}{75} < \frac{1}{74}$	$3 = a + \frac{1}{52}$	$4 = a + > \frac{1}{39} < \frac{1}{38}$
38	$1 = a + \frac{1}{38}a$	$2 = a + > \frac{1}{71} < \frac{1}{70}$	$3 = a + > \frac{1}{50} < \frac{1}{49}$	$4 = a + > \frac{1}{37} < \frac{1}{36}$
36	$1 = a + \frac{1}{36}a$	$2 = a + > \frac{1}{67} < \frac{1}{66}$	$3 = a + > \frac{1}{47} < \frac{1}{46}$	$4 = a + > \frac{1}{35} < \frac{1}{34}$
34	$1 = a + \frac{1}{34}a$	$2 = a + > \frac{1}{63} < \frac{1}{62}$	$3 = a + \frac{1}{44}$	$4 = a + > \frac{1}{33} < \frac{1}{32}$
32	$1 = a + \frac{1}{32}a$	$2 = a + > \frac{1}{59} < \frac{1}{58}$	$3 = a + > \frac{1}{42} < \frac{1}{41}$	$4 = a + > \frac{1}{31} < \frac{1}{30}$
30	$1 = a + \frac{1}{30}a$	$2 = a + > \frac{1}{55} < \frac{1}{54}$	$3 = a + > \frac{1}{39} < \frac{1}{38}$	$4 = a + > \frac{1}{29} < \frac{1}{28}$
28	$1 = a + \frac{1}{28}a$	$2 = a + > \frac{1}{51} < \frac{1}{50}$	$3 = a + \frac{1}{36}$	$4 = a + > \frac{1}{27} < \frac{1}{26}$
26	$1 = a + \frac{1}{26}a$	$2 = a + > \frac{1}{47} < \frac{1}{46}$	$3 = a + > \frac{1}{34} < \frac{1}{33}$	$4 = a + > \frac{1}{25} < \frac{1}{24}$
24	$1 = a + \frac{1}{24}a$	$2 = a + > \frac{1}{43} < \frac{1}{42}$	$3 = a + > \frac{1}{31} < \frac{1}{30}$	$4 = a + > \frac{1}{23} < \frac{1}{22}$
22	$1 = a + \frac{1}{22}a$	$2 = a + > \frac{1}{39} < \frac{1}{38}$	$3 = a + \frac{1}{28}$	$4 = a + > \frac{1}{21} < \frac{1}{20}$
20	$1 = a + \frac{1}{20}a$	$2 = a + > \frac{1}{35} < \frac{1}{34}$	$3 = a + > \frac{1}{26} < \frac{1}{25}$	$4 = a + > \frac{1}{19} < \frac{1}{18}$
18	$1 = a + \frac{1}{18}a$	$2 = a + > \frac{1}{31} < \frac{1}{30}$	$3 = a + > \frac{1}{23} < \frac{1}{22}$	$4 = a + > \frac{1}{17} < \frac{1}{16}$
16	$1 = a + \frac{1}{16}a$	$2 = a + > \frac{1}{27} < \frac{1}{26}$	$3 = a + \frac{1}{20}$	$4 = a + > \frac{1}{15} < \frac{1}{14}$
14	$1 = a + \frac{1}{14}a$	$2 = a + > \frac{1}{23} < \frac{1}{22}$	$3 = a + > \frac{1}{18} < \frac{1}{17}$	$4 = a + > \frac{1}{13} < \frac{1}{12}$
12	$1 = a + \frac{1}{12}a$	$2 = a + > \frac{1}{19} < \frac{1}{18}$	$3 = a + > \frac{1}{15} < \frac{1}{14}$	$4 = a + > \frac{1}{11} < \frac{1}{10}$
10	$1 = a + \frac{1}{10}a$	$2 = a + \frac{1}{15}$	$3 = a + \frac{1}{12}$	$4 = a + > \frac{1}{9} < \frac{1}{8}$
8	$1 = a + \frac{1}{8}a$	$2 = a + > \frac{1}{12} < \frac{1}{11}$	$3 = a + > \frac{1}{10} < \frac{1}{9}$	$4 = a + > \frac{1}{7} < \frac{1}{6}$
6	$1 = a + \frac{1}{6}a$	$2 = a + > \frac{1}{8} < \frac{1}{7}$	$3 = a + > \frac{1}{7} < \frac{1}{6}$	$4 = a + > \frac{1}{5} < \frac{1}{4}$
4	$1 = a + \frac{1}{4}a$	$2 = a + \frac{1}{4}$	$3 = a + \frac{1}{4}a$	$4 = a + \frac{1}{2}a$