ENCYCLOPÉDIE OU *DICTIONNAIRE* RAISONNÉ DES SCIENCES, DES ARTS ET DES MÉTIERS

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DÉ

Die, (*Analysis of chance*.) It is clear that with two *dice* one is able to produce thirty-six different throws; for each of the six faces of the *die* is able to be combined six times with each of the six faces of the other. Similarly with three *dice* one is able to produce 36×6, or 216 different throws: for each of the 36 combinations of the two *dice* can be combined six times with the six faces of the third *die*. Therefore in general with a number of *dice*$= n$, the number of the possible throws is 6^n.

Therefore the odds are 35 against 1 that one won't make a pair of 1, of 2, of 3, of 4, of 5, of 6, with two *dice*. *See* **Rafle**. But one will find that there are two ways to make 3, 3 to make 4, 4 to make 5, 5 to make 6, & 6 to make 7, 5 to make 8, 4 to make 9, 3 to make 10, 2 to make 11, 1 to make 12; this which is apparent by the following table which expresses all 36 combinations.

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

In the first vertical column of this table, I suppose that one of the *dice* falls successively on all its faces, the other *die* always producing 1; in the second column, that one of the *dice* always produces 2, the other producing its six faces, &c. the like numbers are found on the same diagonal. One sees therefore that 7 is the number that it is the most advantageous to wager that one will produce with two *dice*, & that 2 & 12 are those which give the least advantage. If one takes the pain to form also the table of combinations for three *dice*, one will have six tables of 36 numbers each, of which the first will have 3 in the upper left, 13 in the lower right, & the last will have 8 in the upper left, & 18 in the lower right; & one will see by means of the diagonals, that the number of times that the number 8 is able to occur is equal to $6 + 5 + 4 + 3 + 2 + 1$, that is to say to 21; that therefore there are 21 cases out of 216 in order that this number occur, that there are 15 cases to produce 7, 10 for 6, 6 for 5, 3 for 4, 1 for 3; that in order to produce 9 there is a number of combinations $= 5 + 6 + 5 + 4 + 3 + 2 + 1 = 25$; that in order to produce 10 there are $4+5+6+5+4+3 = 27$; that in order to produce 11 there are $3+4+5+6+5+4 = 27$; that in order to produce 12 there are $2+3+4+5+6+5 = 25$; that in order to produce 13 there are $1+2+3+4+5+6 = 21$; that in order to produce 14 there are 15; that in order

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to produce 15 there are 10; that in order to produce 16 there are 6; that in order to produce 17 there are 3; and in order to produce 18, one single combination. Thus 10 & 11 are the two numbers which it is the most advantageous to wager that one will produce with three *dice*, the odds are 27 out of 216, that is to say 1 against 8, that one will produce them; next there is nine or twelve, next there is eight or thirteen, &c.

One is able to determine by a similar method which are the numbers that the odds are the greatest that one will produce with a given number of dice; this which is good to know in several games. *See* **Baraïcus**,[1] **Trictrac**, &c. (*M. d'Alembert*)

[1]This refers to the Boura Heracles. Pausanias, Book VII, 25 §6 reports the following. In Achaia downstream from the town of Boura along the Boura river there is the Herakles in a Grotto who gives oracles with a board and dice. Persons who consulted the oracle first prayed before the statue, selected four dice from a ready supply, and then cast them onto the table. For each possible cast, there was a corresponding interpretation written on the board. The grotto has since been obliterated by earthquakes. The form of the dice and the characters written upon them are unknown.