Second Mèmoire sur la détermination des orbites des planètes et des comètes*

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The new method which I have presented, for the determination of the orbits of planets and of comets, offers two quite distinct parts. I begin by developing, by the aid of my new formulas of interpolation, the variable quantities, especially the geocentric longitude and latitude of the observed star, according to the ascending powers of time; since I substitute the coefficients of the first terms of the developments obtained into some equations which determine the distances from the star to the sun and to the earth, the distance to the sun being first furnished by the resolution of an equation of the first degree. The second of these two problems are resolved promptly without any difficulty, and would furnish the rigorous values of the sought distances, if the values of the coefficients found were themselves exact. It is therefore to render the determination of these coefficients as easy and as exact as it is possible, that one must cling above all.

On another side, the facilities that the new formulas of interpolation present for the determination of which there is question are found considerably augmented when one employs five equidistant observations, or even more generally five observations, of which four, taken two by two, are symmetrically placed on both sides of the middle observation. Then, in fact, the most considerable part of the work, namely the formation of certain numbers which depend uniquely on the epochs in which the observations have been made is completely eliminated, these numbers being able to be immediately furnished by a table similar to the one of page 408,[1] as one will see it after this.

[476] One would simplify therefore notably the solution of the problem, if one would be able to restore the general case to the particular case of which we just spoke. Now a very simple way to arrive there, and to augment at the same time the precision of the calculation, consists in deducing first from the given observations the particular values of the variables corresponding to some equidistant epochs or, at least, to some symmetrically placed epochs on both sides of a middle epoch. The formula of interpolation of Lagrange, from which one would extricate only with great difficulty the general values of the variables, developed into a series ordered according to the ascending powers of time, is, on the contrary, eminently proper to furnish by logarithms the particular values of which it is question. Besides, one particular value corresponding to

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[1] *Translator's note*: See "Mémoire sur la détermination des orbites des planètes et des comètes."

a given epoch will be able to be deduced, in diverse manners, from observations made at some neighboring epochs and combined among them two by two, or three by three, ... If the diverse combinations do not furnish exactly the same value, one will be able to take a mean among the diverse results found, and one will obtain thus a particular value which will be generally much more exact than the values immediately given by the observations themselves.

Formulas relative to the system of five observations, of which four, taken two by two, are symmetrically placed on both sides of a middle observation.

We take for unity the interval of time which will separate the middle observation from the two nearest neighboring observations. Let besides n be the interval of time which will separate the middle observation from the extreme observations. By constructing a table similar to the one of page 408, one will obtain, for the numbers designated by $\alpha, \beta, \gamma, \delta$, the following values:

$$\begin{array}{lllll}
\text{Values of } \alpha \ldots & -\frac{n}{2n+2}, & -\frac{1}{2n+2}, & \frac{1}{2n+2}, & \frac{n}{2n+2}; \\
\text{Values of } \beta \ldots & \frac{n^2}{2n^2+2}, & \frac{1}{2n^2+2}, & \frac{1}{2n^2+2}, & \frac{n^2}{2n^2+2}; \\
\text{Values of } \gamma \ldots & -\frac{1}{4}, & \frac{1}{4}, & -\frac{1}{4}, & \frac{1}{4}; \\
\text{Values of } \delta \ldots & \frac{1}{4}, & -\frac{1}{4}, & -\frac{1}{4}, & \frac{1}{4}.
\end{array}$$

We add that the general values of $\alpha, \beta, \gamma, \delta$, expressed as function of t, will be

$$\alpha = \frac{t}{2(n+1)}, \quad \beta = \frac{t^2}{2(n^3+1)}, \quad \gamma = \frac{t^3 - (n^2 - n + 1)t}{4n(n-1)},$$
$$\delta = \frac{(n^2+1)t^4 - (n^4+1)t^2}{4n^2(n^2-1)}$$

[477] If one puts in particular $n = 2$, one will find the table and the formulas of which I have made use in the determination of the distances of Mercury to the sun and to the earth, namely the table and the formulas of pages 408 and 409.

If one put, on the contrary, $n = 3$, the particular values of $\alpha, \beta, \gamma, \delta$ would be the following:

$$\begin{array}{lllll}
\text{Values of } \alpha \ldots & -\frac{3}{8}, & -\frac{1}{8}, & \frac{1}{8}, & \frac{3}{8}; \\
\text{Values of } \beta \ldots & \frac{9}{20}, & \frac{1}{20}, & \frac{1}{20}, & \frac{9}{20}; \\
\text{Values of } \gamma \ldots & -\frac{1}{4}, & \frac{1}{4}, & -\frac{1}{4}, & \frac{1}{4}; \\
\text{Values of } \delta \ldots & \frac{1}{4}, & -\frac{1}{4}, & -\frac{1}{4}, & \frac{1}{4};
\end{array}$$

while the general values of $\alpha, \beta, \gamma, \delta$ would be

$$\alpha = \frac{t}{8}, \quad \beta = \frac{t^2}{20}, \quad \gamma = \frac{t^3 - 7t}{24}, \quad \delta = \frac{10t^4 - 81t^2}{288}.$$

Finally, if one made $n=4$, the particular values of $\alpha, \beta, \gamma, \delta$ would be the following:

$$\begin{array}{lllll} \text{Values of } \alpha \ldots & -0.4, & -0.1, & 0.1, & 0.4; \\ \text{Values of } \beta \ldots & \frac{8}{17}, & \frac{1}{34}, & \frac{1}{34}, & \frac{8}{17}; \\ \text{Values of } \gamma \ldots & -\frac{1}{4}, & \frac{1}{4}, & -\frac{1}{4}, & \frac{1}{4}; \\ \text{Values of } \delta \ldots & \frac{1}{4}, & -\frac{1}{4}, & -\frac{1}{4}, & \frac{1}{4}; \end{array}$$

while the general values of $\alpha, \beta, \gamma, \delta$ will be

$$\alpha = \frac{t}{10}, \beta = \frac{t^2}{34}, \gamma = \frac{t^3 - 13t}{48}, \delta = \frac{17t^4 - 257t^2}{960}.$$

The particular and general values of $\alpha, \beta, \gamma, \delta$ were thus known, one will obtain without difficulty the sought developments of the variables in series ordered according to the ascending powers of time. Thus, in particular, in order to obtain the geocentric longitude ϕ of the observed star, developed in a similar series, it will suffice to join to the value of ϕ, given by the middle observation, the value of $\Delta\phi$ determined by the formula

$$\Delta\phi = \alpha S \Delta\phi + \beta S' \Delta^2\phi + \gamma S'' \Delta^3\phi + \delta S''' \Delta^4\phi,$$

[478] in which one will substitute the general values of $\alpha, \beta, \gamma, \delta$. Besides the numerical values of the sums

$$S\Delta\phi, \quad S'\Delta^2\phi, \quad S''\Delta^3\phi, \quad S'''\Delta^4\phi$$

will be furnished with those of the differences

$$\Delta\phi, \quad \Delta^2\phi, \quad \Delta^3\phi, \quad \Delta^4\phi,$$

by a new table analogous to the one of page 408; and in order to establish the exactitude of the numbers contained in this new table, it will suffice to be assured that they satisfy, as they must do it, the conditions

$$S\Delta^2\phi = 0, \quad S'\Delta^3\phi = 0, \quad S''\Delta^4\phi = 0.$$

By operating as one just said, one will be able, in the developments of the variables, to conserve the terms proportional to the first four powers of time. The calculations would become simpler, if one neglected the terms proportional to the third and to the fourth power, this which would permit being limited to make use of three observations alone. Then, in fact, γ and δ would vanish. But then also one would obtain, usually, a precision much less great in the results of the calculation.

We observe further that the terms furnished by the formula of interpolation of Lagrange, applied to the determination of particular values of the variables, will be each very small, and, consequently, very easy to calculate, if one is arranged in a manner that the particular values correspond to some epochs very extended from some of the given observations.