SOLUTION D'UN PROBLÈME DE COMBINAISONS

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In how many ways can one effect the product of n factors, equal or unequal? I have resolved this problem, for the case where the factors are unequal, in volume III of this Journal, page 515. Now, if one changes the word *alphabetic* into the word *determined*, and if at the same time one replaces the number of permutations of n different letters, by the number of arrangements of n factors among which there would be α of them equal to a, β equal to b, \ldots, θ equal to t, etc., one will see that my solution is applied completely to the actual problem; so that by designating by Z_{n+1} this number of ways, one will have

$$Z_{n+1} = \frac{n(n+1)\cdots(2n-2)}{1.2.3\ldots\alpha\times1.2.3\ldots\beta\times\cdots\times1.2.3\ldots\theta}.$$

For example, the product of 6 factors equal to a, is able to be effected in 42 ways.

One recalls perhaps that I have treated the case of unequal factors, on the occasion of the decomposition of a polygon into triangles. In the month of November 1838, Mr. Olinde Rodrigues has inserted into this Journal another solution of the same problem.

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