Quelques propriétés des moyens arithmétiques de puissances de quantités positives.*

Jules Bienaymé

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— Mr. Jules Bienaymé gives the statement of some properties of arithmetic means of powers of positive quantities. He remembers first that diverse questions of mechanics and of probabilities have led since a long time to demonstrate that the arithmetic mean of the squares of many quantities is greater than the square of the arithmetic mean of these quantities. One is able to express this theorem in another way, and say that the square root of the mean of the squares of certain quantities is always greater than the mean of these quantities. One sees then more easily that this proposition is only a particular case of another more general, which consists in this that the root of any degree of the arithmetic mean of the powers of the same degree, is always greater than each similar expression in which the degree is inferior. This is that which is indicated algebraically by saying that the value of

$$\left(\frac{c_1a_1^m + c_2a_2^m + \cdots + c_na_n^m}{c_1 + c_2 + \cdots + c_n}\right)^{\frac{1}{m}}$$

increases or diminishes always with m.

This proposition and many consequences which are deduced from it on the magnitudes relative to the means of powers are susceptible to numerous applications analogous to those which depend simply on the mean of the squares. One concludes from it, for example, that

$$\frac{a_1 + a_2 + a_3 + \cdots + a_n}{n} < (a_1^{a_1}a_2^{a_2}a_3^{a_3}\cdots a_n^{a_n})^{\frac{1}{a_1+a_2+\cdots a_n}}$$

One knows already that on the contrary

$$\frac{a_1 + a_2 + \cdots + a_n}{n} > (a_1a_2a_2\cdots a_n)^{\frac{1}{n}}$$

*Translated by Richard J. Pulskamp, Department of Mathematics & Computer Science, Xavier University, Cincinnati, OH. June 20, 2010