Calcul des probabilités.*

Jules Bienaymé

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Mr. Jules Bienaymé communicated some remarks on the manner in which persons little habituated to observe nature would be able to understand that which one calls *the constancy of the mean results* of observations. According to him, scholars, to whom the significance of these words is familiar, should not for that neglect to take from time to time the subject of short explications, especially when their researches must end by entering into the mass of ideas of a circulation a little general.

"The words *constancy of the mean results* express that it becomes more and more probable, in measure as the observations multiply themselves, to see to be reproduced some means sensibly equal, when it has been already gathered a great number of similar means little different among them. This reproduction is not assured at all, although very probable, and no educated observer would regard as contrary to the order of nature that it had not taken place during one or many series of new experiences. It would be only there an extraordinary fact. But for the everyone, among which today one repeats often that the mean results of great numbers are fixed, are constants, it would not be astonishing that they take in this regard a false idea. — They would be able to imagine, for example, that if it rains much during three months in sequence, and if the quantity of rain already fallen has attained very nearly the mean annual quantity observed during a long time, the nine other months of the year could not lack being in an extreme drought. They would suppose that the compensation is of absolute necessity, because the mean annual result is a quantity very nearly fixed, that it has not varied sensibly for long years, and that since then the contrary event has no longer an infinitely small probability, so to say. — It would be, one sees it, an error of the same kind as that which ruled a short time ago among the persons who played in the lottery. They had learned by the writings of the geometers how much it is possible that a designated number not exit in a series of numerous drawings: thus there is odds more than 999 against 1, that a series of 121 drawings, or of 605 numbers, will contain this designated number at least one time. The players concluded from it, when already a number had remained in the wheel during more than 100 drawings, that there were by this fact only one enormous probability of seeing it exit in one of the drawings most forthcoming. The popular

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works on the probabilities indicate this error. But the paralogism on which it rests is not explicated cleanly: and it is for that without doubt that it has persisted with force during the fifty years of the existence of the lottery. Each time that the absence of a number is remarked in a series of drawings as little it be prolonged, the receipts of the public treasury are increased.

"Since the paralogism of which there is concern is able to be attached to some important results, there is yet utility to demonstrate well from it the deficiency. One knows that we believe so much more in a future event, if it takes part of a more certain set, and if the other part already realized in this example was for us less probable isolation before arriving. It is to this that it is reduced, in last analysis, all conjectural reasoning. Our mind evaluates rapidly the probability of the complete event, composed of the arrived facts and of the awaited facts, as if the set of these facts were yet to come: it evaluates next separately the probability of the arrived facts, as if they were likewise yet futures, next it divides the first probability by the second; this which returns to reverse the fraction which expresses this second probability, and to make it the multiplier of the first. The quotient, or rather the product thus obtained, is for us the probability of the awaited facts, deduced from the observed facts. This logical march could deceive only on one point; it is the evaluation more or less exact of the original probabilities of the facts considered, and consequently the increase of the set of facts of which it is necessary to take account. The players of lottery transgressed not at all by the use of the form of reasoning; but they are mistaken on the choice of the probabilities to compare. It is quite true that there is an excessive probability, that it is nearly certain that a series of 121 drawings will contain a designated number; but it is under the condition that this number will occupy any place in the order of the drawings. Now one is no longer subject to this condition when one assigns the place of these numbers in the last drawings. It was not therefore this great probability of the exit in any one place out of 605 places, that it was necessary to take for the term of comparison. The set of the arrived facts and of the future facts being, for example, 110 drawings consummated without seeing the given number, and 11 drawings awaited, in which it must exit at least one time; the probability of this composite event must enter alone into the mental calculation that a player had to make. It would be then easily to be convinced that this last probability, far from being near to unity, as this $(\frac{999}{1000})$ from the mean result of any place, had on the contrary only a weak value hardly superior to 1 out of 2500. The probability of the arrived part, that is of 110 drawings without the given number, being of the same order of magnitude $\frac{1}{1276}$, the quotient of the one by the other $\frac{1276}{2500}$ could give only a probability quite mediocre to the non-arrived part, or to the exit of of the given number at least one time in 11 drawings. Here this probability, which is reduced near to $\frac{51}{200}$, is precisely equal, as one knows, to that which one would have had before the 110 first drawings; so that the past facts could have no influence on the facts to come; and, indeed, there exists none of them when the possibilities of the events are known as in a lottery.

"Although the possibilities of the natural events are unknown, the vice of the reasoning is absolutely the same when one considers the quantity of rain which is fallen in three months of a year, and that which is able to fall in the following nine months. Effectively, the great probability of the annual mean result near invariable supposes essentially that the distribution of water throughout the year is able to be any. There is

therefore no comparison to make of this probability with that which must be referred to a special distribution of the rain. In this last case determined, the composite event of which one must calculate the probability consists in a great rain during three months, followed by a great drought. And it will be to divide this probability, whatever it be, by that of a great rain of three months calculated isolated. These two unknown probabilities are not formed directly in the summaries of meteorological observations, but one is able to deduce them; then alone one will know if there is any reason to believe in a next drought when the mean quantity of annual rain comes to be very nearly exhausted, or else if then even new rains remain possible. One would find without doubt that it is this last case which the meteorological tables indicate, although the mean quantity of water fallen each year is to here of a remarkable constancy.

"The preceding just employed in order to render the error manifest is a general application to all the results of observations which deviate from the constant means of great numbers of observations. If one wished also to give some attention, it would serve to dissipate more of the prejudice widespread in the world, even among the men who have received that which one qualifies ordinarily of a good education."

—In the sequence of this communication, Mr. Babinet cites a curious experience on chances, which has been made by a person to whom he had indicated a way certain to win in the lottery; he had counseled to him to play the extract during a great number of years, by taking always at each drawing the 45 older numbers. Wishing to be convinced by himself of the certitude of the proposed way, the person of whom there is concern makes to raise it from all the books of the lottery since its origin to 1822, and supposing that it had been played during all this lapse of time against the bank, conformably to the opinion that he had received, he calculated that which a sensible combination would have produced finally. The result found was that he would have obtained 5.32 numbers for each double drawing, whence it is easy to conclude that the advantage had ended by being on his side.